TEST ITEM DEPENDENCE OF SEVERAL OBLIQUE FACTOR SOLUTIONS

By

JOHN PELHAM BAILEY, JR.

A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF
THE UNIVERSITY OF FLORIDA
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA
1969

ACKNOWLEDGEMENTS

I should like to acknowledge with gratitude and thanks the help of dozens of people who have advised, counseled, listened, and reacted during my years of graduate work. However, space permits explicit recognition for:

Dr. Wilson H. Guertin who, as my mentor for over four years and as Chairman of my supervisory committee, was chiefly instrumental in guiding my emerging interests and suggesting the area for this study;

Dr. Benjamin Barger, Dr. Robert Jester, and Dr. Bobby Cage who, as members of my supervisory committee, performed their functions with skill and understanding;

Dr. Evelyn Wenzel who, as my master's advisor, led me by example through my first years in graduate work;

Mr. Jon Seidel and Mr. John Soderstrum, whose analytic and programming skills I partook extensively of; and

Dr. George H. Dunteman, without whose example, understanding, and knowledge I might never have trod this path.

TABLE OF CONTENTS

		Page
ACKNOWLEDGEMENTS		ii
LIST OF TABLES		v
I	GENERAL CONSIDERATIONS	1
	Background	2
	Simple Structure and Positive Manifold	4
	Reference and Primary Factors	6
	Rotation to Simple Structure	7
	Automatic Machine Rotation Methods	9
	Factor Invariance	14
	The Problem	16
II	REVIEW OF THE LITERATURE	18
III	METHOD	21
	The Correlation Matrix	21
	The Initial Factor Solution	21
	Rotation Methods	24
	Scheme of Analysis	26
	Indices of Stability	31
	Stability Comparisons	33
IV	RESULTS AND DISCUSSION	35
	Stability of Factor Loadings	35
	Stability of Factor Intercorrelations	38

Page

Discussion 41

Simple Structure 44

V CONCLUSIONS 49

Purpose of the Study 49

Significance of the Results 51

BIBLIOGRAPHY 53

APPENDIX 55

BIOGRAPHICAL SKETCH 149

LIST OF TABLES

TABLE		Page
3.1	INTERCORRELATIONS OF TWENTY-FOUR PSYCHOLOGICAL TESTS	22
3.2	CENTROID MATRIX FOR 24 PSYCHOLOGICAL TESTS	23
3.3	VARIMAX SOLUTION FOR TWENTY-FOUR PSYCHOLOGICAL TESTS	25
3.4	TEST NUMBER AND FACTOR LOADING PATTERN	27
3.5	TEST BATTERY MODIFICATIONS	29
3.6	LOADINGS FOR TEST 6 AND TEST 20 IN ALL CONDITIONS	32
4.1	FACTOR LOADINGS RMSA ANALYSIS OF VARIANCE SUMMARY TABLE	36
4.2	FACTOR LOADINGS RMSP ANALYSIS OF VARIANCE SUMMARY TABLE	37
4.3	FACTOR INTERCORRELATIONS RMSA ANALYSIS OF VARIANCE SUMMARY TABLE	39
4.4	FACTOR INTERCORRELATIONS RMSP ANALYSIS OF VARIANCE SUMMARY TABLE	40
4.5	CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 3	46
4.6	CODED SUMMARY OF ROTATED FACTOR LOADINGS - NINE CONDITIONS	48
A - 1	VARIMAX REFERENCE FACTORS - 20 TEST BATTERY	56
A - 2	VARIMAX REFERENCE FACTORS - CONDITION 1	57
A - 3	VARIMAX REFERENCE FACTORS - CONDITION 2	58
A - 4	VARIMAX REFERENCE FACTORS - CONDITION 3	59
A - 5	VARIMAX REFERENCE FACTORS - CONDITION 4	60
A - 6	VARIMAX REFERENCE FACTORS - CONDITION 5	61

TABLE Page

A - 7 VARIMAX REFERENCE FACTORS - CONDITION 6 62

A - 8 VARIMAX REFERENCE FACTORS - CONDITION 7 63

A - 9 VARIMAX REFERENCE FACTORS - CONDITION 8 64

A - 10 MAXPLANE PRIMARY FACTORS - 20 TEST BATTERY 65

A - 11 MAXPLANE PRIMARY FACTORS - CONDITION 1 66

A - 12 MAXPLANE PRIMARY FACTORS - CONDITION 2 67

A - 13 MAXPLANE PRIMARY FACTORS - CONDITION 3 68

A - 14 MAXPLANE PRIMARY FACTORS - CONDITION 4 69

A - 15 MAXPLANE PRIMARY FACTORS - CONDITION 5 70

A - 16 MAXPLANE PRIMARY FACTORS - CONDITION 6 71

A - 17 MAXPLANE PRIMARY FACTORS - CONDITION 7 72

A - 18 MAXPLANE PRIMARY FACTORS - CONDITION 8 73

A - 19 OBLIMIN PRIMARY FACTORS - 20 TEST BATTERY 74

A - 20 OBLIMIN PRIMARY FACTORS - CONDITION 1 75

A - 21 OBLIMIN PRIMARY FACTORS - CONDITION 2 76

A - 22 OBLIMIN PRIMARY FACTORS - CONDITION 3 77

A - 23 OBLIMIN PRIMARY FACTORS - CONDITION 4 78

A - 24 OBLIMIN PRIMARY FACTORS - CONDITION 5 79

A - 25 OBLIMIN PRIMARY FACTORS - CONDITION 6 80

A - 26 OBLIMIN PRIMARY FACTORS - CONDITION 7 81

A - 27 OBLIMIN PRIMARY FACTORS - CONDITION 8 82

A - 28 BINORMAMIN PRIMARY FACTORS - 20 TESTS 83

A - 29 BINORMAMIN PRIMARY FACTORS - CONDITION 1 84

A - 30 BINORMAMIN PRIMARY FACTORS - CONDITION 2 85

A - 31 BINORMAMIN PRIMARY FACTORS - CONDITION 3 86

A - 32 BINORMAMIN PRIMARY FACTORS - CONDITION 4 87

TABLE Page

A - 33 BINORMAMIN PRIMARY FACTORS - CONDITION 5 88

A - 34 BINORMAMIN PRIMARY FACTORS - CONDITION 6 89

A - 35 BINORMAMIN PRIMARY FACTORS - CONDITION 7 90

A - 36 BINORMAMIN PRIMARY FACTORS - CONDITION 8 91

A - 37 OBLIMAX PRIMARY FACTORS - 20 TESTS 92

A - 38 OBLIMAX PRIMARY FACTORS - CONDITION 1 93

A - 39 OBLIMAX PRIMARY FACTORS - CONDITION 2 94

A - 40 OBLIMAX PRIMARY FACTORS - CONDITION 3 95

A - 41 OBLIMAX PRIMARY FACTORS - CONDITION 4 96

A - 42 OBLIMAX PRIMARY FACTORS - CONDITION 5 97

A - 43 OBLIMAX PRIMARY FACTORS - CONDITION 6 98

A - 44 OBLIMAX PRIMARY FACTORS - CONDITION 7 99

A - 45 OBLIMAX PRIMARY FACTORS - CONDITION 8 100

A - 46 SIMPLE LOADINGS PRIMARY FACTORS - 20 TESTS 101

A - 47 SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 1 102

A - 48 SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 2 103

A - 49 SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 3 104

A - 50 SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 4 105

A - 51 SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 5 106

A - 52 SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 6 107

A - 53 SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 7 108

A - 54 SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 8 109

A - 55 VARIMAX ROTATION - FACTOR I 110

A - 56 VARIMAX ROTATION - FACTOR II 111

A - 57 VARIMAX ROTATION - FACTOR III 112

A - 58 VARIMAX ROTATION - FACTOR IV 113

TABLE Page

A - 59 MAXPLANE ROTATION - FACTOR I 114

A - 60 MAXPLANE ROTATION - FACTOR II 115

A - 61 MAXPLANE ROTATION - FACTOR III 116

A - 62 MAXPLANE ROTATION - FACTOR IV 117

A - 63 OBLIMIN ROTATION - FACTOR I 118

A - 64 OBLIMIN ROTATION - FACTOR II 119

A - 65 OBLIMIN ROTATION - FACTOR III 120

A - 66 OBLIMIN ROTATION - FACTOR IV 121

A - 67 BINORMAMIN ROTATION - FACTOR I 122

A - 68 BINORMAMIN ROTATION - FACTOR II 123

A - 69 BINORMAMIN ROTATION - FACTOR III 124

A - 70 BINORMAMIN ROTATION - FACTOR IV 125

A - 71 OBLIMAX ROTATION - FACTOR I 126

A - 72 OBLIMAX ROTATION - FACTOR II 127

A - 73 OBLIMAX ROTATION - FACTOR III 128

A - 74 OBLIMAX ROTATION - FACTOR IV 129

A - 75 SIMPLE LOADINGS ROTATION - FACTOR I 130

A - 76 SIMPLE LOADINGS ROTATION - FACTOR II 131

A - 77 SIMPLE LOADINGS ROTATION - FACTOR III 132

A - 78 SIMPLE LOADINGS ROTATION - FACTOR IV 133

A - 79 MAXPLANE ROTATION PRIMARY FACTOR CORRELATIONS 134

A - 80 OBLIMIN ROTATION PRIMARY FACTOR CORRELATIONS 135

A - 81 BINORMAMIN PRIMARY FACTOR CORRELATIONS 136

A - 82 OBLIMAX PRIMARY FACTOR CORRELATIONS 137

A - 83 SIMPLE LOADINGS PRIMARY FACTOR CORRELATIONS 138

A - 84 CODED SUMMARY OF ROTATED FACTOR LOADINGS - 20-TEST BATTERY 139

TABLE Page

A - 85 CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 1 140

A - 86 CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 2 141

A - 87 CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 3 142

A - 88 CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 4 143

A - 89 CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 5 144

A - 90 CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 6 145

A - 91 CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 7 146

A - 92 CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 8 147

A - 93 MEANS, STANDARD DEVIATIONS, AND RELIABILITY COEFFICIENTS 148

Chapter I

GENERAL CONSIDERATIONS

As a general scientific method, factor analysis has come a long way from the groping concepts of Galton and Spearman to the broad scope theory construction of Cattell. Through the methodological contributions of men like Thurstone and Holzinger, factor analysis has reached a degree of acceptance among both mathematical statisticians and theoreticians in psychology. In recent years high speed computers have permitted and encouraged more use of its powerful techniques. However, it can be argued that the idiosyncrasies of the method are not sufficiently well known and understood to justify such wide and indiscriminant use as has been made of factor analysis. For example, until recently little was known about the sampling distribution of a rotated factor loading (Cliff and Pennell, 1967).

One specific aspect of this which has not been explored is the test item dependence of the automatic oblique machine solutions. Kaiser (1958) published the first widely used machine rotation solution, and he demonstrated that his varimax solution provides relatively constant loadings for a group of tests no matter what battery those tests are imbedded in. The varimax solution, however, produces factors which correlate .00 with one another--they are independent. Since about 1960, numerous machine rotation routines have been produced which seek to solve the rotation problem by permitting the angles

between the factors to be oblique. The test item dependence of these solutions has not been determined.

This dissertation will deal with this very important and basic methodological determination of test item dependence over six different analytic and semi-analytic machine rotation methods, five of which permit oblique solutions. It should provide an important contribution to the knowledge about these methods which are exerting such a strong force in modern factor analysis.

Background

"A factor problem starts with the hope or conviction that a certain domain is not so chaotic as it looks" (Thurstone, 1947, p. 55). Factor analysis seeks, as a general scientific method, to determine the underlying order of the domain, as represented by the specific variables under consideration, to assist in the formulation of general scientific concepts about the domain. Factor analysis does not require a formal hypothesis as to the nature of this underlying order; this order is disclosed without first being postulated. The necessary and sufficient assumption to factor analytic methodology is simply that the underlying principles are fewer than the number of data variables involved in the study. Factor analysis is a method designed to reveal these principles.

Factor analysis is exploratory in character and, as most writers make clear, its most useful function is on the frontier or leading edge of a science. Its particularly useful role is to provide a map of the domain in question so that the more exact methods of experimentation and statistical analysis may work with more appropriate and generally useful data and criteria. Thus, factors are many things to many people

depending on context: functional unities, parameters, intervening constructs, abilities, and even causes. They will be referred to here solely by the reasonably context-free word "factors."

Factor analysis may be considered a system of sequential statistical analyses, a system which consists of three distinct steps. Typically, first a correlation matrix is formed of inter-relationships among variables over a sample or group of subjects. This is then operated on by one of a variety of methods (Harman, 1967, mentions more than ten distinct types) to produce a multiple-factor matrix which is considered only an initial arbitrary factor solution. A final matrix is achieved by transforming this initial solution into another form which, it is hoped, permits a more useful interpretation of the factorization. This transformation process is most often referred to as "rotation." Among the various rotation procedures there is first the basic choice between orthogonal and oblique factors, and then the choice between several alternative transformation methods. The form of the final solution is typically a matrix in which the rows represent variables, the columns relate to factors, and the matrix elements, which are usually called loadings, represent correlations between variables and factors in the sense of being orthogonal projections on axes. While the relationship expressed by these matrix elements may be more complex when the factors themselves are correlated, the general term factor loading may still be used.

All these considerations lie in the realm of *common*-factor analysis, i.e., factorization of a correlation matrix which is of a rank less than the number of variables. This means that the diagonal elements of the correlation matrix are not unities (1.0) but rather

some lesser number which is an estimate of the common variance (communality) of each variable. Often R^2 is used as an estimate. Factorization of a matrix with unities in the diagonal is properly referred to as "principal components analysis" and should not be subjected to rotation. Rotation of components is common practice (Glass and Taylor, 1966); but in the search for common dimensions or constructs to aid in charting a domain it does not seem useful to cram unknown amounts of unique variance into the structure. Factor analysis attempts to deal only with common variance, excluding the error variance and specific variance of the general linear model.

Simple Structure and Positive Manifold

Except for simple linear prediction from components, the factor analysis problem demands that there shall be a psychologically meaningful interpretation of the factors derived in terms of which the set of variables can be understood. The initial factor matrix is arbitrary, i.e., it is a function of the method of factoring. There are dozens of these methods, and seldom does any factoring method immediately result in a psychologically meaningful resolution. A final factor solution becomes more successful to the degree that it is meaningful and plausible as an interpretation of underlying order. The Simple Structure concept is one (and so far probably the best) way of achieving meaningful final factor solutions. As Thurstone (1947) puts it, "The principles of simple structure are fundamental in making factor analysis a scientific method rather than merely a method of statistical condensation, and it is therefore one of the central themes in factorial theory" (p.319). It is one thing to condense, but quite another to achieve meaningful simplification.

Our search for underlying order can also be expressed as the search for a set of descriptive categories (factors) with which to represent our variables in the simplest possible terms. Simple structure as a criterion was developed through this basic idea of finding "the smallest number of parameters for describing each test" (Thurstone, 1947, p. 333). This implies solving for a unique solution, which is a basic aim of simple structure. Thurstone's five criteria (Thurstone, 1947, p. 335) by which meaningful and unique structure can be achieved may be expressed as follows (Horst, 1965, p. 385):

1. For each factor, only a small number of variables will have high loadings, and the remainder will have low loadings.

2. Each variable will have appreciable loadings on only a few factors.

For a given pair of factors,

3. A number of variables will have low loadings on both factors.

4. Others will have high loadings on one of the factors and low loadings on the other.

5. Still other variables will have high loadings on the second factor but low loadings on the first.

A factor matrix which has been transformed to meet these criteria may be referred to as a simple structure factor loading matrix.

The positive manifold criterion is yet another aspect of transformation to simple structure. Simply stated, it means that if there were no negative correlation coefficients in the correlation matrix there should be no negative loadings in the final factor solution. This principle goes hand in hand with the simple structure criteria in practice; and any assessment of the degree that simple structure is achieved by a transformed solution necessarily includes its consideration.

Reference and Primary Factors

The final solution, or simple structure factor loading matrix, may be either a V matrix or a P matrix. Only in the orthogonal case are they identical. The V matrix represents orthogonal projections of variables onto axes (factors) which are normal to their hyperplanes. The effect of this reference factor matrix V is to show the composition (in terms of variables) of the factors. This could mean that a primary aim of simple structure for the V matrix would be the first criterion.

On the other hand, the P matrix, or primary factor matrix, is different. P axes are located at the intersections or traces of hyperplanes, and loadings of variables on these axes are determined by oblique rather than orthogonal projections of variables. Geometrically, the line of orthogonal projection of a variable onto a reference axis intersects a corresponding primary axis at an angle, and the point of intersection of the P axis is the variable's P factor loading. It may be equal to or larger than its corresponding V matrix loading, and this difference is a function of the angular separations of reference axes. P matrix loadings can be greater than 1.0; hence another reason why they should not be thought of as correlation coefficients. The primary factor matrix P shows the composition (in terms of factors) of the variables. This could mean that a primary aim of simple structure for the P matrix would be the second criterion--factorial simplicity of items (variables) across factors.

There is a mathematical relationship between the V and P matrices (Harman, 1967), and many oblique machine rotation procedures give both the V and P matrices because the V matrix is often a calculational step on the way to a P solution. The V matrix is said to be particularly

useful for understanding the nature of factors in exploratory work, while the P matrix helps to understand the nature of the variables or tests. Both can be used to assess adequacy of simple structure; but because the P matrix shows both row and column simplicity, it would seem the better for establishing factor locations. There is a continuing problem in the literature, however, as to which matrix is being reported because of inconsistent terminology and general unfamiliarity with the distinctions. Harman (1967) tries to inject clarity into the situation by suggesting the Reference Structure (V) elements be called loadings, and that Primary Pattern (P) elements be called coefficients. The literature remains confusing, however, when Fruchter (1954) calls V factors "simple axes" and Jennrich and Sampson (1966) call P elements "simple loadings."

The present study deals only with P matrices because they seem more useful. However, for ease of communication, all these P matrix elements will be called loadings.

Rotation to Simple Structure

The process of attaining simple structure probably is most easily understood in graphic terms. Variables are plotted on all pairs of factors in two-dimensional graphs. The factors are treated as axes which are then literally rotated (some writers prefer to think of the variables being rotated) until desired simplicity of structure (projections on axes) is achieved. This process involves extensive labor, many intuitive and subjective decisions, and a great deal of algebraic calculation. The whole meaning of a solution can be changed by a comparatively slight modification in the locations of several

factors. It can immediately be seen that the desired scientific objectivity of a solution across data analysts cannot be met because there are no universally accepted ground rules for graphic rotation. Further, there is no assurance that a unique final solution has been reached. This is not to say, however, that plotting and rotation in iterative cycles are passé. For finer looks at a domain structure by a skilled factor analyst there are as yet unsurpassed, largely because the criteria set up by Thurstone are descriptive rather than mathematical definitions of simple structure. Thurstone's criteria were intuitive, and an intuitive solution seems to make the best ultimate use of them.

Since the advent of computers, the search has been on for an automatic machine rotation procedure which would attain the best simple structure possible in a factor study. Generally speaking, these analytic methods rotate the arbitrary factor matrix factors into trial positions until a mathematical function of all the test item loadings on each is maximized (or minimized). A mathematical function derived from Thurstone's five criteria is established and this function is treated as if it were the definition of simple structure. Through extensive computational effort (years for men but minutes for computers) the arbitrary matrix is transformed into a matrix whose function values are optimized in accordance with the method. If the function will not permit the attainment of simple structure in the problem, subsequent graphical analysis might if such a structure is possible in that situation. Apparently, at this time in the development of machine rotation procedures, the only way to assess the degree of simple structure attained is to make plots and subjectively decide. However, Cattell (1966) points out,

> The objectivity of a resolution resides in the statistical significance check on the position finally reached, regardless of the instrumentalities which led to the assumed point of completion. . . . A statistical test of simple structure . . . applied just as with any other conclusion is appropriate . . . but any estimate of probability based on which particular variables fall in the hyperplanes proves computationally to be very costly (p. 188).

Neither this nor plotting is often done because the modern machine methods seem to present reasonably good solutions. They have relieved the experimenter of the need to be present while his data are being analyzed, and they permit remarkably good and useful structures and patterns to result. These results are objective in the sense that they can be replicated by any other person using the same formulae.

Automatic Machine Rotation Methods

Six well-known and frequently used machine methods of rotation to simple structure will be compared in this study. The selection is arbitrary on the part of the author, and it is based on their being representative in the field as well as for their having certain unique characteristics. The following explanations of the procedures are taken from Harman (1967) and Eber (1966).

Varimax (Kaiser, 1958) places emphasis on simplifying the columns of the factor matrix by maximizing the variance of a factor's squared loadings over all tests. This results in greater simplicity of a factor because its loadings are forced toward unity or zero. Related to the entire factor matrix, the varimax criterion requires the maximization of the sum of all the factors' simplicities. Successive trial locations of all possible $m(m-1)/2$ pairs of factors determine where this function stabilizes at a maximum value. Kaiser found that there was systematic bias in the solution due to the implicit weighting of

tests by their communalities. This bias is removed by equally weighting the tests for rotation through the procedure of dividing each row by the square root of the test's communality. The original weighting is restored after rotation. This normalization process, often referred to as "Kaiser normalization," is seen in many analytic procedures by the "h" elements appearing in their criterion formulae. Harman expresses the varimax criterion as:

$$V = n \sum_{p=1}^{m} \sum_{j=1}^{n} (b_{jp}/h_j)^4 - \sum_{p=1}^{m} \left(\sum_{j=1}^{n} b_{jp}^2/h_j^2 \right)^2 .$$

The varimax method is restricted to an orthogonal solution, and this satisfies the simple structure criteria rather well in the case of orthogonal or nearly orthogonal structure. The remaining five procedures to be considered remove this orthogonality restriction and permit greater flexibility in the form of the solution. An orthogonal solution may result from their application should the "best" solution for that problem tend toward orthogonality. Correlations among the primary factors and among the reference axes are normally a part of the output from computer programs of these oblique procedures.

Oblimax (Pinzka and Saunders, 1954) is one of several similar methods devised independently at about the same time, and it attempts to arrive at an analytic determination of simple structure by maximizing the number of large and small factor loadings at the expense of the medium-size loadings. The criterion to be maximized is expressed as the ratio of the sum of the fourth powers of the loadings on a factor, to the squared sum of the squared loadings on that factor. Successive trial locations of pairs of factors lead to the ultimate maximization

of the criterion for this method, as expressed by Harman:

$$K = \sum_{j=1}^{n} \sum_{p=1}^{m} v_{jp}^4 \Bigg/ \left(\sum_{j=1}^{n} \sum_{p=1}^{m} v_{jp}^2 \right)^2 .$$

Oblimin is a class of oblique rotation methods which use a minimizing criterion. Three separately recognizable criteria are within this class. The quartimin criterion, developed by Carroll in 1953, involved the minimization of the sum of cross products between factor pairs of squared factor loadings. The covarimin criterion came about in relaxing Kaiser's orthogonality restriction in varimax, and it involved the minimization of the covariance of the squared elements of each pair of factors. Harman reports (p. 325) that almost invariably the covarimin is biased toward orthogonality while the quartimin is biased toward too highly correlated factors. A biquartimin criterion was proposed by Carroll (1957) as a compromise between these two extremes, and this was achieved by generalizing from the two criteria to a single oblimin criterion with a parameter (γ) which can be varied to result in the solution desired. Harman expresses the oblimin criterion as:

$$B = \sum_{p<q=1}^{m} \left[\sum_{j=1}^{n} (v_{jp}^2/h_j^2)(v_{jq}^2/h_j^2) - \gamma \sum_{j=1}^{n} v_{jp}^2/h_j^2 \sum_{j=1}^{n} v_{jq}^2/h_j^2 \right] = \min.$$

When $\gamma = 1$, the covarimin criterion results; when $\gamma = 0$, the quartimin criterion results; and when $\gamma = .5$, the biquartimin criterion results. Any other value between zero and unity may be used, but there will be appropriate variations in the resulting factor structures. The biquartimin is recommended as best for general use.

In an attempt to avoid the undetermined parameter in the oblimin procedures, which must be decided upon by the experimenter, the *binormamin* procedure was developed (Kaiser and Dickman, 1959). The criterion to minimize in this procedure is expressed by Harman as:

$$D = \sum_{p<q=1}^{m} \left[\sum_{j=1}^{n} (v_{jp}^2/h_j^2)(v_{jq}^2/h_j^2) \Big/ \left(\sum_{j=1}^{n} v_{jp}^2/h_j^2 \right) \left(\sum_{j=1}^{n} v_{jq}^2/h_j^2 \right) \right] = \min.$$

Harman indicates that the relative superiority of either the oblimin (biquartimin) or the binormamin method seems to lie more with the nature of the data than with any theoretical difference. Neither seems to do sufficiently well in all cases so as to exclude use of the other. Of particular interest for the present study is the fact that the best comparative results for binormamin have been with Holzinger and Harman's Twenty-Four Psychological Tests data (Holzinger and Harman, 1941), which will be used in the present study.

All of the previously discussed methods deal with the optimization of some function of the elements of the reference structure (V matrix) loadings. This can be viewed as an intermediate step in the attainment of simple structure in the P matrix. Transformation from V to P is easily accomplished on a computer, but this involves extra computational steps. Jennrich and Sampson (1966) developed a procedure called *simple loadings* which is analogous to the oblimin class, in that it produces a class of solutions based on the value of an arbitrary parameter and uses a minimizing criterion. However, the simple loadings procedure works directly on P matrix elements rather than V. Harman refers to the method as "direct oblimin," and for comparison with the oblimin criterion he expresses this new criterion (b's represent P elements) as:

$$F(P) = \sum_{p<q=1}^{m} \left(\sum_{j=1}^{n} b_{jp}^2 b_{jq}^2 - \frac{\delta}{n} \sum_{j=1}^{n} b_{jp}^2 \sum_{j=1}^{n} b_{jq}^2 \right).$$

Harman reports extensive experimentation with various values for the parameter δ, but he concludes that there is no simple way to relate the oblimin parameter values and the simple loadings parameter values. He has found that both $\delta = -.5$ and $\delta = .0$ give results quite similar to oblimin (biquartimin) for the Holzinger and Harman data. A "standard" value of zero is given as a recommended value in the UCLA Biomedical Computer Programs version of this procedure (BMD X72), and this program also permits Kaiser normalization if desired. Harman suggests that the simple loadings procedure may be superior to the oblimin class not only because it works directly on the P matrix, but also because the parameter may take on values considerably beyond zero and unity.

Cattell (1966) expresses criticism of the analytic solutions because they do not get directly at the heart of simple structure, which he feels is the maximization of the number of tests falling in hyperplanes (p. 188). This is what plotting and algebraic rotation do; and the development first by Cattell and Muerle (1960) and later by Eber (1966) of the *maxplane* procedure produced this in a semianalytic fashion on a computer. Maxplane starts off with a normalization procedure and then makes trial rotations of single factors of factor pairs. Factor position is determined by the location at which the number of tests falling in the hyperplane of the factor being rotated is maximized. The later development by Eber weighted this hyperplane count largely on the basis of the factor loading of a test on the irrelevant factor of the pair to prevent factor collapse. In an iterative manner the maxplane procedure tries

locations for both factors of every possible factor pair in the problem and selects final locations where the total weighted hyperplane count is at a maximum.

Factor Invariance

There are at least two broad aspects of the concept of factor loading variance which need to be considered. One of the most noted and vocal authorities in the field of factor analysis, R. B. Cattell, focuses on the invariance of factor identification from study to study. A consideration of the matrix algebra in the factoring problem will show that the arbitrary factor solution (the second step in the factor analysis problem) is not a unique solution for the matrix being factored. The equation solved in arriving at this matrix solution is the finding of a matrix which when multiplied by its transpose will equal the original correlation matrix being factored. As long as orthogonality between factors is maintained this will be true for any number of factor solutions for a given correlation matrix. Rotation to simple structure with the positive manifold criterion has as one very important purpose the creation of a final solution which is unique to the problem--unique in the sense that it is "the" final solution for the factorization of a particular correlation matrix. Granted that the unique simple structure position has been generated for a particular study, Cattell's concern is about that structure's invariance across different samples and different tests purporting to measure the same thing. This is a concern for the determination of pyschological traits, pervasive dimensions, or generally occurring factors which invariably are a part of the map of a domain. This is a very complex question

which he covers in some detail (Cattell, 1966, p. 184ff) and which is his concept of the *raison d'être* of factor analysis as a scientific method.

The other aspect of factor invariance is imbedded in the method itself, and particularly in the transformation process. What assurance is there to the investigator that a unique simple structure position has been generated for the particular tests under study? What are the influences on factor loadings which cause them to vary? These questions are prior to Cattell's grander concern. Thurstone (1947) defines two aspects of this type of invariance with his general dictum that, ". . . the factorial description of a test must remain invariant when the test is moved from one battery to another which involves the same common factors (p. 361). *Metric invariance* simply means that the numerical values of factor loadings must not vary, and *configurational invariance* relates to the invariance of the pattern of loadings as well as the intercorrelations of the oblique axes, or the configuration of the final solution. Thurstone suggests that if we are to attempt to ascertain a set of generally useful parameters for a domain (Cattell's concern) then we must be concerned about the metric and configurational invariance of the structures we have discovered.

Putting it another way, the simple structure obtained must be unique for that type of variable or test and sample of subjects so that it is relatively invariant no matter what other tests constitute the total battery. In fact, Kaiser (1958) suggests that Thurstone was really on the trail of factor invariance in this sense when he formulated the five simple structure criteria. This would mean that factor invariance is the ultimate criterion of the rotation problem, and that analytic methods

which rotate to optimize a mathematical function of a matrix based on a factorial simplicity concept may not be aimed at the correct end. Horst (1965, p. 495) says that invariance is the objective of simple structure, and that simple structure is justified in terms of invariance. The concern for the invariance properties of a solution is well stated by Kaiser (1958):

> This invariance property would seem to be of greater significance than the numerical tendencies . . . to define mathematically the doctrine of simple structure. Although factor analysis seems to have many purposes, fundamentally it is addressed to the following problem. Given an (infinite) domain of psychological content, infer the internal structure of this domain on the basis of a sample of n tests drawn from the domain. The possibility of success from such inference is obviously dependent upon the extent which a factor derived from a particular battery or sample of tests approximates the corresponding unobservable factor in the infinite domain. If a factor is invariant under changing samples of tests, i.e., shows factorial invariance (Thurstone, p. 360-361), there is evidence that inferences regarding domain factors are correct (p. 195).

Harman (1967) clarifies the relationship between simple structure and the invariant properties of a final solution. The latter do not ascribe any greater "psychological meaning" to the solution; but they do reflect a greater likelihood of a particular solution portraying population factors, or of an observed set of factors portraying the corresponding domain factors. This inferential matter has an obvious bearing on the need to establish the sampling distribution of rotated factor loadings.

The Problem

When Kaiser published his varimax criterion procedure in 1958, he showed that the varimax solution ". . . is invariant under changes in composition of the test battery" (p. 195). He did this algebraically

for a general two-factor case and empirically for the four-factor data which the present study will use. He alluded to insuperable mathematical difficulties in generalizing from two to *m* factors. His results showed that varimax loadings were not overly dependent on the test item make-up of the rotated battery.

The oblique transformation methods often are preferred over the orthogonal varimax method because they permit orthogonality to assert itself when present, rather than restricting the solution to orthogonality. Computer procedures produce oblique solutions quickly, in comparison to the slow graphic and algebraic methods of only a few years ago. To what degree, if at all, do their rotated loadings depend on which tests are included in sampling?

If we are to depend on and use these oblique machine methods so extensively, there should be a demonstration of their factor invariance. The purpose of this study is to determine if there is any difference in the factor loading invariance characteristics of several machine rotation methods. This will be done by using identical systematic changes in test battery composition carried into rotation by each method. The dependence of the solutions of each method on the various test compositions of the battery will be assessed by examining the changes in the tests' P matrix loadings, as they are included in various batteries. This should establish whether or not these methods are capable of insuring that ". . . the factorial description of a test [as determined by a particular transformation method] remains invariant when the test is moved from one battery to another which involves the same common factors."

Chapter II

REVIEW OF THE LITERATURE

The concerns of this dissertation developed from the discussion by Harman (1967) of the factor invariance problem in the context of the varimax solution. Harman makes it clear when discussing varimax that because of its demonstrated factor invariance, ". . . varimax factors obtained in a sample will have a greater likelihood of portraying the universe varimax factors" (p. 313). Elsewhere on the subject of invariance he states, "Invariance as used in the context of rotation to a preferred multiple-factor solution requires that the factorial description of a variable must remain the same when it is a part of one battery or another which involves the same common factors . . . . The varimax method is proposed by Kaiser as the factorially invariant solution" (p. 272). However, in subsequent extensive and detailed discussions of numerous oblique transformation methods, from which much material was drawn for Chapter I of this study, he does *not* follow this with a parallel statement for any of the oblique transformation procedures.

Each of the papers which describe the oblique transformation methods to be compared was reviewed (Carroll, 1957; Cattell and Muerle, 1960; Eber, 1966; Jennrich and Sampson, 1966; Kaiser and Dickman, 1959; Saunders, 1961). None reveals either discussion or demonstration of metric invariance. There are some general articles which touch on the

question (Cattell, 1965; Ferguson, 1954; Guilford and Zimmerman, 1963; and Thompson, 1962). While the title of Meredith's (1964) article "Notes on Factorial Invariance" is suggestive, he is concerned there with the mathematics of invariance of solutions over subsamples rather than invariance of solutions over changes in test battery.

The point is that varimax seems to be the only analytic rotation solution designed for automatic machine rotation which has been examined in the literature for dependence upon test items. The emphasis throughout the relevant literature is on the invariance of concepts across samples. Cattell (1966) offhandedly puts it that, "Naturally, we are talking about studies on different subjects . . ." (p. 191).

Horst (1965), in his matrix algebra treatment of factor analysis, agrees that metric factor invariance is essential to a good rotation solution. But he states, "One of the chief difficulties with most of the analytical [oblique and orthogonal] methods which have been developed is that they are greatly influenced by the particular selection of variables . . ." (p. 418). The oblique machine rotation methods are more likely to attain or surpass the high degree of stability of factor loadings shown by Kaiser for varimax. Commenting specifically on the varimax solution, Horst says, "The possibility of achieving relative invariance of transformed factor loadings . . . is more remote than if more general [oblique] transformation procedures were available" (p. 420). Cliff and Pennell (1967), in an excellent study of the influences on the characteristics of factor loadings, state, ". . . if the orthogonality restriction of the transformation was relaxed, the resulting sampling errors [of loadings] would be smaller . . ." (p. 325). This implies along with Horst that oblique procedures would permit better inferences of domain factor structure.

The Cliff and Pennell paper is part of the continuing attack on determining the sampling distribution of a rotated factor loading. Cliff and Hamburger (1966) conclude that its standard deviation is about the same as that of a product-moment correlation coefficient ($1/\sqrt{N}$). Cliff and Pennell try to pin this down by studying the effects of the number of factors, number of tests, communality, and factorial complexity of variables. They conclude that within one factoring and one rotation procedure, ". . . communality rather than loading size as such is the important determiner of stability . . . . The important characteristic of a matrix of a given size is the communalities of the tests. Higher communalities mean not only greater stability for the loadings of specific tests but also lead to stronger factors which mean that the stability of *all* the loadings is improved" (p. 325).

Cliff and Pennell cite an obvious consequence to their methodological approach, which is that generalization of their results to empirical factor studies using different methods of factoring or rotation is tenuous. They conclude, therefore, that there is a future to the study of metric invariance of factor loadings in the continuing search for approximate tests of significance for the loadings.

The present study is designed to add to the body of literature in this regard by holding constant the method of factoring, while using different transformation methods on a systematically varied initial factor matrix. It should serve to show the instability of factor loadings as a function of rotation method. Each method will be used over a wide range of test battery sampling with the communality of the tests included in the battery systematically varied.

Chapter III

METHOD

The Correlation Matrix

The basic battery of tests to be used in this study of the test item dependence of six automatic machine rotation procedures is Holzinger and Harman's Twenty-Four Psychological Tests (Holzinger and Harman, 1941). These data were gathered in 1934-36 by Holzinger and Swineford on 145 seventh and eighth grade school children in a suburb near Chicago. They have been used extensively since then in numerous factor analytic studies and demonstrations of method. So much work has been done with them, in fact, that they represent a sort of classic in factor analysis. The basic statistics of these data (means, standard deviations, and reliability coefficients) are presented in Table A-93, and the correlation matrix is presented in Table 3.1. The complete descriptive data for all the tests are given in Holzinger and Harman (1941), Appendix B.

The Initial Factor Solution

The initial common-factor solution of the correlation matrix of Table 3.1 is the four-factor orthogonal centroid solution shown in Table 3.2. R^2 was used as the communality estimate. Although Harman (1967) gives five initial solutions from among the many possible, with from four to seven factors extracted, this centroid solution was

TABLE 3.1

INTERCORRELATIONS OF TWENTY-FOUR PSYCHOLOGICAL TESTS*

Test	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1																								
2	32																							
3	40	32																						
4	47	23	31																					
5	32	29	25	23																				
6	34	23	27	33	62																			
7	30	16	22	34	66	72																		
8	33	16	38	39	58	53	62																	
9	33	20	18	33	72	71	69	53																
10	12	06	-08	10	31	20	25	29	17															
11	31	15	09	11	34	35	23	30	28	48														
12	31	15	14	16	22	10	18	27	11	59	43													
13	49	24	32	33	34	31	35	40	28	41	54	51												
14	13	10	18	07	28	29	24	25	26	17	35	13	20											
15	24	13	07	13	23	25	17	18	25	15	24	17	14	37										
16	41	27	26	32	19	29	18	29	24	12	31	12	28	41	33									
17	18	01	18	19	21	27	23	26	27	29	36	28	19	34	35	32								
18	37	26	21	25	26	17	16	25	21	32	35	35	32	20	33	34	45							
19	27	11	31	14	19	25	23	27	27	19	29	11	26	21	19	26	32	36						
20	37	29	30	34	40	44	45	43	45	17	20	25	24	30	27	39	26	30	16					
21	37	31	17	35	32	26	31	36	27	41	40	36	43	18	23	35	15	36	33	41				
22	41	23	25	38	44	39	40	36	48	16	30	19	28	24	25	28	27	32	34	46	37			
23	47	35	38	34	44	43	41	50	50	26	25	35	38	24	26	36	29	27	30	51	45	50		
24	28	21	20	25	42	43	44	39	42	53	41	41	36	30	17	26	33	41	37	37	45	38	43	

*Holzinger and Harman, 1941

Decimal points omitted in this and most subsequent tables.

TABLE 3.2

CENTROID MATRIX FOR 24 PSYCHOLOGICAL TESTS*

Tests	Factors I	II	III	IV	h^2
1	608	-116	300	-250	536
2	372	-119	207	-135	214#
3	427	-220	262	-155	323#
4	477	-211	206	-184	348#
5	668	-306	-344	108	670
6	661	-337	-258	216	664
7	652	-396	-384	124	745
8	662	-225	-153	-060	516
9	664	-394	-240	308	749
10	462	455	-365	-136	572
11	569	397	-208	-063	529
12	484	360	-149	-388	537
13	508	130	-099	-402	558
14	442	199	-013	293	321
15	407	170	146	266	287
16	523	077	300	076	375
17	492	317	082	338	464
18	547	307	248	072	460
19	452	125	129	111	249#
20	612	-174	128	004	421
21	601	114	080	-171	410
22	608	-144	145	136	430
23	691	-164	129	116	534
24	654	151	-150	-003	473
Sum a_{ij}^2	7.637	1.600	1.145	1.001	11.385

*Holzinger & Harman, 1941, p. 191. #Omitted in rotational schemes

selected because it is the one which has been used by many developers of rotational schemes to demonstrate their methods (Kaiser, 1958; and Eber, 1966, among others). This can provide a degree of comparability of final solutions not available otherwise. Also, the pattern of varimax rotated loadings from this initial solution is particularly useful.

Table 3.3 shows the varimax solution for the centroid matrix of Table 3.2. Note in Table 3.3 that tests 2, 3, 4, and 19 have particularly low communalities as well as generally low factor loadings. Their reliability coefficients are given in Table A-93 of the Appendix. Tests 2, 3, and 4 have their highest loadings on factor III, which is already well defined by tests 1, 20, 21, 22, and 23. Test 19 has its highest loading on factor IV, which is also well defined by five other tests. Exclusion of these four low communality tests would not substantially affect the multiple-factor solution to this problem. The reason for their exclusion will be made clear.

Rotation Methods

The computer rotation methods used in this study are varimax, oblimax, binormamin, oblimin (biquartimin), maxplane, and simple loadings. The varimax program used is the UCLA Biomedical Computer Programs version (BMD 03M). The oblimax and binormamin programs used are presently in use at the University of Illinois. They are adaptations of the original Saunders and the original Kaiser and Dickman procedures, and versions are running on the University of Florida IBM System/360-50. The oblimin (biquartimin) program used is an adaptation of Carroll's method by Geertsma. The maxplane program

TABLE 3.3

VARIMAX SOLUTION FOR TWENTY-FOUR PSYCHOLOGICAL TESTS*

Tests	Factors I	II	III	IV	h^2
1	14	19	67	17	536
2	10	07	43	10	214
3	15	02	54	08	232
4	20	09	54	07	348
5	75	21	22	13	670
6	75	10	23	21	664
7	82	16	21	08	745
8	54	26	38	12	516
9	80	01	22	25	749
10	15	70	-06	24	572
11	17	60	08	36	529
12	02	69	23	11	537
13	18	59	41	06	558
14	22	16	04	50	321
15	12	07	14	50	287
16	08	10	41	43	375
17	14	18	06	64	464
18	00	26	32	54	460
19	13	15	24	39	249
20	35	11	47	25	421
21	15	38	42	26	410
22	36	04	41	36	430
23	35	21	57	22	534
24	34	44	22	34	473
Sum a_{ij}^2	3.50	2.44	3.08	2.36	11.385

*Harman, 1967

came from the University of Georgia version of the Eber (1966) modification. It was adapted to run on the University of Florida's computer installation. A simple loadings procedure has recently become available as part of the BMD package (BMD X72) and this was adapted to run at Florida. For this method, the value zero was used for δ, the parameter in the criterion, on the advice of Jennrich, and the Kaiser normalization procedure was used. Although the main thrust of this study concerns the oblique methods, orthogonal varimax solutions are included to permit some additional comparisons as well as to refine the judgement of factor invariance made by Kaiser for his method.

Scheme of Analysis

In this study each of the machine rotation procedures is required to rotate sample variations on the centroid factor solution of Table 3.2. There are eight variations (or *conditions*, as they will hereafter be called), plus the original centroid solution, which are imposed on the test battery. Each condition represents the exclusion of certain of the twenty-four tests of the basic battery from rotation, and each condition should be viewed as a test battery in itself. The effect of these conditions is to move each test from battery to battery involving the same common factors. If the factor loadings for a test remain relatively constant over these changing batteries (conditions), then factor invariance has been demonstrated; and the method used will then be shown to be insensitive to test battery variations.

Table 3.4 shows the tests arranged with the test numbers appearing in columns corresponding to the factors on which they have the

TABLE 3.4

TEST NUMBER AND FACTOR LOADING PATTERN

(Communalities in Parentheses)

Rank by Communality	Factors I	II	III	IV	Sum of h^2
1	9 (.75)	10 (.57)	1 (.54)	17 (.46)	2.32
2	7 (.74)	13 (.56)	23 (.53)	18 (.46)	2.29
3	5 (.67)	12 (.54)	22 (.43)	16 (.38)	2.02
4	6 (.66)	11 (.53)	20 (.42)	14 (.32)	1.93
5	8 (.52)	24 (.47)	21 (.41)	15 (.29)	1.69
Sum of h^2	3.34	2.67	2.33	1.91	10.25
Percent of Total h^2	32	26	23	19	

highest loadings when rotated to the varimax criterion (Table 3.3). Notice that factor I consists of tests with high communality, factor IV consists of tests with low communality, and that factors II and III fall somewhere in between. The exclusion of tests 2, 3, 4, and 19 has permitted Table 3.4 to have the symmetric appearance that it does. The configuration displayed in this table is maintained for descriptive purposes generally invariant over all rotation samples.

According to Thurstone (1947, p. 363), there must be enough tests representing a factor in a test battery to determine a factor. This is not less than two tests per factor, which defines a common factor; but only two might make the factor location uncertain. The plan of this study in imposing selective conditions of test exclusion on the battery takes this into account by having five, four, three, and two tests representing the factors--an increasingly harsh imposition on the rotation procedure to locate factors. This sampling varies the battery test composition without, in most cases, seriously affecting the battery factor composition or the factor composition of the tests in the battery.

In addition to the simplicity of excluding various tests from rotation in order to vary the test battery actually carried into rotation, careful consideration was given to the communality of the tests excluded in each condition. This was directly the result of Cliff and Pennell's (1967) assertion regarding the importance of communality and therefore factor strength on the stability of factor loadings. Table 3.5 shows the plan of the eight test battery modifications in which certain tests are excluded. Careful examination of Table 3.5 in conjunction with Table 3.4 will reveal the systematic varying of test communality and factor strength in each condition beyond the initial "All" condition, in which 20 tests are carried into rotation. It was

TABLE 3.5

TEST BATTERY MODIFICATIONS

Condition	Factor	Sum of h^2	Percent of Total	No. of Tests	Tests Excluded	Description of Condition
All	I	3.34	32	5	None	Balanced matrix
	II	2.67	26	5	None	of strong and
	III	2.33	23	5	None	weak tests and
	IV	1.91	19	5	None	factors.
		10.25	100	20		
1	I	1.49	21	2	5,6,8	Exclusion of
	II	1.67	23	3	11,24	weak tests in
	III	1.92	28	4	21	strong factors.
	IV	1.91	28	5	None	
		6.99	100	14		
2	I	3.34	43	5	None	Exclusion of
	II	2.20	28	4	24	weak tests in
	III	1.50	19	3	20,21	weak factors.
	IV	0.92	11	2	14,15,16	
		7.96	100	14		
3	I	3.34	46	5	None	Exclusion of
	II	2.10	29	4	10	strong tests in
	III	1.26	17	3	1,23	weak factors.
	IV	0.61	8	2	16,17,18	
		7.31	100	14		

TABLE 3.5 (Continued)

Condition	Factor	Sum of h^2	Percent of Total	No. of Tests	Tests Excluded	Description of Condition
4	I	1.18	18	2	5,7,9	Exclusion of
	II	1.54	24	3	10,13	strong tests in
	III	1.74	28	4	1	strong factors.
	IV	1.91	30	5	None	
		6.42	100	14		
5	I	1.85	33	3	7,9	Exclusion of
	II	1.54	27	3	10,13	2 strongest
	III	1.26	22	3	1,2,3	tests in each
	IV	0.99	18	3	17,18	factor.
		5.64	100	12		
6	I	2.16	33	3	6,8	Exclusion of
	II	1.67	25	3	11,24	2 weakest
	III	1.50	23	3	20,21	tests in each
	IV	1.30	19	3	14,15	factor.
		6.63	100	12		
7	I	1.18	33	2	5,7,9	Exclusion of
	II	1.00	28	2	10,12,13	3 strongest
	III	0.83	22	2	1,22,23	tests in each
	IV	0.61	17	2	16,17,18	factor.
		3.62	100	8		
8	I	1.49	32	2	5,6,8	Exclusion of
	II	1.13	25	2	11,12,24	3 weakest
	III	1.07	23	2	20,21,22	tests in each
	IV	0.92	20	2	14,15,16	factor.
		4.61	100	8		

the exclusion of tests 2, 3, 4 and 19 which permitted the balanced exclusions which the conditions represent.

It was anticipated that should any of the six methods severely break down and fail to achieve satisfactory simple structure for any of the conditions imposed, the experimenter could pinpoint what the situation was at that time with respect to all aspects of battery composition and perhaps find parallel problems with the other procedures under the same condition. Unlike a Monte Carlo approach, this systematic manipulation insures that a reasonably full gamut of progressively harsh situations is imposed for each of the machine methods to test its robustness, as well as to permit cross-method comparisons of equal situations without having to rely on principles of random distribution and selection.

Indices of Stability

Carrying this four-factor problem into rotation nine times for each of six methods produced 54 final solution rotated factor matrices. With the exception of varimax, these were transformed to P matrices since we were looking primarily at the factor composition of tests. These matrices are contained in the Appendix as Tables A-1 through A-54. In order to look at the individual test's loadings for each factor under these nine conditions of test battery composition, the factor loadings were re-arranged in such a way as to make their variability across conditions more evident. Table 3.6 consists of excerpts from Table A-55, which is the factor I loadings of the 20 tests when they have been rotated to the varimax criterion in all nine conditions.

TABLE 3.6

Varimax Loadings for Test 6 and Test 20 in All Conditions

	Conditions									
Test	All	1	2	3	4	5	6	7	8	RMSA	RMSP
6	76		77	75	77	75		75		0089	0118
20	38	34		38	46	40		41		0434	0563

Stability would be demonstrated by the loadings for a test not substantially changing, as is the case with test 6 which was included in five of the eight battery conditions. On the other hand, for the same number of loadings, test 20 is considerably more variable in its values for factor loadings. This is empirically indicated by the values in the columns headed RMSA and RMSP. RMSA represents the root-mean square of the sum of the squared differences between each loading in a test row and the loading for that test in the "All" condition. RMSP is the root-mean square of the sum of the squared differences between all possible pairs in a test row, including the "All" condition. Thus in this example, five differences go into RMSA and fifteen go into RMSP. The permutations of tests to define conditions for each method resulted in two tests having seven loadings for each of four factors, twelve tests having six, and six tests having five loadings per factor.

Both the RMSA and the RMSP statistics were carried into the final analysis and reported because there seems to be adequate rationale for using either as an indication of the stability of the loadings. The "All" condition is the largest of the nine batteries carried into rotation, has the best and most balanced test and factor representation, and is the standard battery from which the other eight are derived.

It seems reasonable to judge stability by the differences from this criterion condition. However, there are at most only six elements entering into the calculation of each RMSA. On the other hand, by considering the differences between all possible pairs of loadings for a test on a factor, a minimum of ten elements are summed in the RMSP statistic. This latter also emphasizes the stability-across-conditions aspect of the study, in that the loadings should be stable among various conditions as well as with respect to a criterion condition.

The process of laying out the successive factor loadings and the RMSA and RMSP figures was carried out for every test on every factor in every rotation method. Tables A-55 through A-78 show these arrangements. The same processes of calculating variability were done for the factor intercorrelations for each of the five oblique methods, and these appear in Tables A-79 through A-83.

Stability Comparisons

It can be seen that each method's rotation problems were identical in that the same eight conditions (plus the initial "All") were imposed for each method. This means that 20 RMSA and 20 RMSP values were computed for each factor, or 80 for each method. Taken as an aggregate, each method's 80 indices of factor loading stability (either RMSA or RMSP) can be considered an index of that method's ability to cope with all the conditions imposed on the test battery carried into rotation.

The fixed-effects, one-way analysis of variance with equal replications is an appropriate technique to test the null hypothesis of no differences among the six methods in test loading stability over these conditions. The dependent variable is either the RMSA or the RMSP index

for each test on each factor. Having replicates in each cell of the design equal to 80 makes the analysis quite robust. Similar analyses of variance, but with only six replicates per cell, were conducted on both the RMSA and RMSP statistics resulting from the intercorrelation coefficients among the primary factors for the five oblique methods.

Since planned independent comparisons are not a part of the design of this study, and since it is a misuse of the *t*-test to make comparisons suggested by the data if they were not initially planned, a different method to compare each rotation method with each other must be used. Duncan's Multiple-Range Test (Steele and Torrie, 1960) was used to compare the methods' mean RMSA and RMSP indices after the analyses of variance. Using this test, the probability of finding a significant difference between any two means when the corresponding true means are equal is less than or equal to the significance level chosen. For this study that level is $p = .01$ (two tail).

Chapter IV

RESULTS AND DISCUSSION

Stability of Factor Loadings

There is a significant difference among the methods of rotation in their ability to achieve invariance of factor loadings across the conditions of test battery composition imposed on them. A low mean RMSA or RMSP statistic would indicate relative invariance or stability of factor loadings, and varimax is the best in this regard with a mean RMSA of .0360 and a mean RMSP of .0433. On the other end of the scale lies oblimin (biquartimin) with mean RMSA and RMSP values almost five times as large--.1538 and .1943. The analyses of variance conducted over the 80 RMSA and 80 RMSP indices for each of the six rotation methods are summarized in Tables 4.1 and 4.2, both of which show an F statistic significant beyond the .01 level. This leads to rejection of the null hypothesis of no difference in the stability of factor loadings across the rotation methods.

The application of Duncan's Multiple-Range Test (Tables 4.1 and 4.2) to the mean RMSA and RMSP indices for each pair of methods also shows that although varimax has the lowest mean in both cases, there is statistically no difference between it and either simple loadings, binormamin, oblimax, or maxplane. The analysis using RMSA (Table 4.1) is slightly different in delineating differences among methods than the analysis using RMSP (Table 4.2). The overlap in non-significant

TABLE 4.1

Factor Loadings

RMSA Analysis of Variance Summary Table

Source of Variation	df	Sum of Squares	Mean Square	F
Among Methods	5	0.8863	.178	46.50**
Within Methods	474	1.8017	.004	
Total	479	2.6880		

Duncan's Multiple-Range Test Summary Table

Ranked Means

Varimax	Simple Loadings	Binormamin	Oblimax	Maxplane	Oblimin (biq)
.0360	.0476	.0560	.0944	.1254	.1538

Note:

Any two means underscored by the same line are not significantly different at the .01 level.

** Significant at or beyond the .01 level.

TABLE 4.2

Factor Loadings

RMSP Analysis of Variance Summary Table

Source of Variation	df	Sum of Squares	Mean Square	F
Among Methods	5	1.340	0.268	53.60**
Within Methods	474	2.583	0.005	
Total	479	3.923		

Duncan's Multiple-Range Test Summary Table

Ranked Means

Varimax	Simple Loadings	Binormamin	Oblimax	Maxplane	Oblimin (biq)
.0433	.0575	.0667	.1018	.1416	.1943

Note:

Any two means underscored by the same line are not significantly different at the .01 level.

** Significant at or beyond the .01 level.

pairwise differences (ranked means underscored by the same line) is less using the RMSP statistic; but overall the relative ability of binormamin, oblimax, and maxplane may be considered about equal. The clear fact emerging from this comparison of means for methods is that varimax and simple loadings are on the stable, low variance end, while oblimin (biquartimin) is significantly different from them and on the unstable, high variance end.

Stability of Factor Intercorrelations

The Holzinger and Harman data provide an essentially oblique factor problem, or one in which the oblique methods should be used (rather than the orthogonal methods) to explore the factor structure of the battery or of the domain it represents. The mean absolute factor intercorrelation for the problem when all 20 tests are carried into rotation by the five oblique methods considered is .433, which corresponds to an average angle between factors of about 64°.

There is a significant difference among the oblique methods in their ability to produce stable factor intercorrelation coefficients across the various test batteries used as conditions in which the factors appear. The simple loadings method is the least variable in this regard, with a mean RMSA of .0707 and a mean RMSP of .0947. Maxplane is on the more unstable end with a mean RMSA of .1882 and a mean RMSP of .2351. The analyses of variance conducted over the 6 RMSA and 6 RMSP indices for the factor intercorrelations produced by each oblique method are summarized in Tables 4.3 and 4.4. The F statistics shown are significant beyond the .01 level, indicating that this null hypothesis of no difference among the methods must also be rejected.

TABLE 4.3

Factor Intercorrelations

RMSA Analysis of Variance Summary Table

Source of Variation	df	Sum of Squares	Mean Square	F
Among Methods	4	.0665	.0166	6.64**
Within Methods	25	.0628	.0025	
Total	29	.1293		

Duncan's Multiple-Range Test Summary Table

Ranked Means

Simple Loadings	Binormamin	Oblimax	Oblimin (biq)	Maxplane
.0707	.0847	.1230	.1757	.1882

Note:

Any two means underscored by the same line are not significantly different at the .01 level.

** Significant at or beyond the .01 level.

TABLE 4.4

Factor Intercorrelations

RMSP Analysis of Variance Summary Table

Source of Variation	df	Sum of Squares	Mean Square	F
Among Methods	4	.1044	.0261	5.61**
Within Methods	25	.1139	.0046	
Total	29	.2183		

Duncan's Multiple-Range Test Summary Table

Ranked Means

Simple Loadings	Binormamin	Oblimax	Oblimin (biq)	Maxplane
.0947	.1142	.1538	.2347	.2351

Note:

Any two means underscored by the same line are not significantly different at the .01 level.

** Significant at or beyond the .01 level.

The application of Duncan's Multiple-Range Test to these data (Tables 4.3 and 4.4) shows slightly less range between highest and lowest mean RMSA and RMSP values than with the factor loadings data. In the table of ranked means, maxplane is now the method on the high end and oblimin is next highest. Otherwise, the order of the ranked means is the same as that for the factor loadings analyses. The general configuration of the overlapping non-significant differences is the same, with simple loadings on the stable, low end and both maxplane and oblimin on the unstable, high end. Again, binormamin and oblimax lie about equal to each other in the mid-range.

Discussion

In order to assess what these results may mean to an experimenter who must choose between various oblique methods of rotation, it is perhaps useful to reflect on the stability measures used in these analyses of variance.

There appears to be no useful or substantial difference between the results of analyses using either the RMSA or the RMSP statistics. The RMSP has slightly greater within-methods variance associated with it as would be expected, but this does not appreciably affect the results. Both have been reported in this study simply to show their virtually identical results in spite of their different sampling rationales.

The analysis of the factor loadings themselves would provide a mean factor loading and a variance of factor loadings for each method over all conditions imposed. It would do the same for the factor intercorrelations. These statistics would perhaps have some direct meaning for the reader, but they would not be getting at the heart of the matter, which is stability: changes in the particular test's

loadings over the various conditions. Taking either the root-mean square of the sum of squared pairwise differences of absolute values of test loadings for a single test over all conditions (RMSP), or taking the root-mean square of the sum of the squared differences of loadings from a criterion condition (RMSA), seems to get at this more directly.

Each of the 80 RMSAs and 80 RMSPs (20 for each of four factors) is a direct indication of the stability of a test's factor loadings as determined by a particular automatic rotation method. The higher the statistic, the lower the stability. The less that a test's loadings change as the test is included in the various batteries carried into rotation by a method, then the lower will be the test's index for that method. The less that all 20 tests' loadings change the lower will be that method's mean index. A low mean index for a method indicates high stability for that method.

The analyses of variance simply test to determine if all methods provide equally stable solutions over all test battery conditions. Tables 4.1 and 4.2 clearly show that they do not with respect to factor loadings. Tables 4.3 and 4.4 indicate that neither do they with respect to factor intercorrelations. Finally, because various tests compose the batteries and make them different from condition to condition, high stability would indicate relative independence of methods and test items in batteries. Varimax and simple loadings unequivocally share this characteristic on both factor loadings and factor intercorrelations. Binormamin might be included on the basis of its better showing in the analyses of factor intercorrelations.

In the analyses of the loadings it seems clear the oblimin (biquartimin) method has some trouble at some point in the conditions imposed. Digging behind the summary results of Tables 4.1 and 4.2 reveals why oblimin appears so different from the other methods. The answer lies in the test battery carried into rotation in condition 3. Examination of Tables 3.4 and 3.5 shows that condition 3 excludes the stronger tests of the weaker factors, those tests which tend to identify and locate the factors. Factor I has its full complement of tests present in this 14-test battery while the main tests which define factor IV, the weakest factor, are excluded. Factor I becomes overpowerfully dominant. Notice that nearly half (46%) of the communality carried into rotation is on factor I. The Kaiser normalization procedure, which is a part of oblimin (biquartimin), should permit it to cope with the problem, but all the methods except oblimin (biquartimin) were able to locate primary factors II, III, and IV with a fair degree of success. This can be seen by the condition 3 primary factor matrices for each method in the tables in the Appendix as well as by the coded rotated factor matrices later to be discussed in this chapter. Table A-22 shows that the oblimin (biquartimin) rotation of the condition 3 test battery resulted in the collapse of factor IV, represented by only two weak tests, onto factor III, which is shown by the III-IV correlation coefficient of .969. Factor I emerged as a general factor almost completely independent of the remaining factors, as can be seen by its factor intercorrelations of only -.047, .005, and .035. The effect of this factor collapse on the stability of loadings for oblimin (biquartimin) is evident in Tables A-63 through A-66; and the effect on the stability of the primary factor intercorrelations is evident in Table A-80.

It seems obvious that oblimin (biquartimin) is not sufficiently robust to cope with the test-factor-battery communality combination of condition 3, which is probably not an uncommon situation to be encountered. A good transformation method should be able to reveal the location of the weaker factors even if strong dominance by one factor is encountered. The other methods fare quite well by comparison in this regard. Perhaps the oblimin method with another arbitrary parameter (covarimin or quartimin) would do better, but the biquartimin is the normal standard which was put into comparison with other methods' normal parameters.

Examination of Tables A-55 through A-83 (loadings and intercorrelations stability tables) does not reveal any unusual or salient reasons why the maxplane and oblimax methods should be as relatively unstable as they appear. Even the factor collapse by oblimin (biquartimin) in condition 3, which resulted in factor correlations of nearly zero and unity, was not sufficient to make its mean RMSA or RMSP for correlations greater than maxplane. Although maxplane and oblimax are quite different in criterion, there seems to be a general variability in factor location for both of these methods which becomes significant when viewed in a comparative study.

Simple Structure

The relative ability of the methods to achieve stability of their factor loadings and their factor intercorrelations has been assessed and discussed. Certain methods have been shown to be superior in this regard. However, the other side of the coin in factor rotation problems relates to the goodness of simple structure achieved by a method.

When this study was originally designed it was planned to accept each method's definition of simple structure as the best it could do under the imposed circumstances, and proceed to look only at the dependence of the factor loadings and factor intercorrelations on the test item content of the various sample test batteries. But no factor analyst can simply look at a factor matrix and pass on without assessing its clarity of solution. At this point in the development of factor analytic methodology this is largely a subjective decision based on the rules for simple structure by Thurstone which have been earlier set forth. To make this assessment easier by washing out minor differences, a coding scheme was developed for presenting factor loadings. While this might not be considered strictly appropriate for evaluation of P matrices, it is extremely helpful and revealing.

Table 4.5 is a duplicate of Table A-87. It has been brought forward into this chapter to serve as an example as well as to shed further light on the problem that oblimin (biquartimin) revealed itself as having. Table 4.5 summarizes the ability of each method to achieve simple structure in condition 3 by combining into one table in coded form the six rotated primary factor matrices for this condition of test battery composition.

Good simple structure would be seen in a coded table such as Table 4.5 as a maximum number of low (<.26) loadings and a minimum number of moderate (.26 - .40) loadings. It can be seen that varimax did not do very well in this regard, probably because of the orthogonal restriction being applied to this oblique situation. The fact that oblimin (biquartimin) failed in condition 3 is dramatically seen here. The simple loadings method can be judged the best for this condition,

TABLE 4.5

Coded Summary of Rotated Factor Loadings - Condition 3

Method	Frequency of Occurence				
	Blank	+	++	+++	++++
Varimax	33	10	6	5	2
Maxplane	38	2	8	4	4
Oblimin (biq)	11	3	20	16	6
Binormamin	38	5	7	5	1
Oblimax	37	5	5	5	4
Simple Loadings	40	5	5	3	3

Coding Scheme:

.000 - .259	Blank
.260 - .399	+
.400 - .599	++
.600 - .799	+++
.800 - .999	++++

on the basis of its high number (40) of low loadings. But maxplane is equally satisfactory because of its much lower number (2) of moderate (+) loadings.

Examination of the coded rotated loadings for all test battery conditions (Tables A-84 through A-92) shows that binormamin is superior in five of the nine test batteries carried into rotation, and it fails to come in at least second only in condition 3. Oblimax is superior twice (conditions 2 and 8), and maxplane and simple loadings are superior once each in conditions 1 and 3 respectively.

Table 4.6 summarizes these findings over all rotation samples. The expected overall superiority of binormamin is apparent; but now the relative merit of oblimax, simple loadings, and maxplane emerges more clearly. These three are about equal in the number of low loadings, but maxplane has substantially fewer moderate (+) loadings, 34 as compared to 41 for oblimax and 59 for simple loadings. Thus Table 4.6 would confirm the binormamin method as best among the six in achieving simple structure, with maxplane second.

TABLE 4.6

Coded Summary of Rotated Factor Loadings - Nine Conditions

Method	Frequency of Occurence				
	Blank	+	++	+++	++++
Varimax	271	64	76	42	11
Maxplane	307	34	58	41	24
Oblimin (biq)	278	59	60	48	19
Binormamin	318	36	57	40	13
Oblimax	308	41	50	32	33
Simple Loadings	308	59	46	34	17

Coding Scheme:

.000 - .259	Blank
.260 - .399	+
.400 - .599	++
.600 - .799	+++
.800 - .999	++++

Chapter V

CONCLUSIONS

Purpose of the Study

The purpose of this study was to determine if there is any difference in the test item dependence of several automatic machine rotation procedures for factor analysis. The question was generated as the result of several considerations which combined to make it a necessary study.

Thurstone (1947) stated that the factor composition of a test should remain invariant when the test is moved from one battery to another in which the same common factors exist. This would also mean that the test composition of the common factors should remain relatively invariant under these conditions.

The standard error of a product-moment correlation coefficient $(1/\sqrt{N})$ is used to estimate the approximate standard error of rotated loadings, but there is insufficient information available at the present to describe fully the sampling distribution of rotated factor loadings.

It is known that the automatic machine rotation methods vary in their ability to resolve factor structure. The ability of these methods to produce invariant solutions, in the sense considered by Thurstone, seems to be a valid consideration in exploring the influences on factor loadings. Previous research with only one rotation method (Cliff and Pennell, 1967) isolated some major influences; the present study using several methods intends to refine this area.

Studies using factor analysis as a tool and purporting to reveal the structure of a domain are appearing with increasing frequency in the literature as the transformation procedures, once so expensive and time consuming, are rapidly being made available through fast and easy-to-use computerized procedures. This situation prompts an even more strenuous search for the influences affecting rotated factor loadings. If the method of transformation itself is a contaminating influence, this should be known. Inferences regarding domain structure should be based on a least-variant solution if possible.

The purpose of this study came sharply into focus when a search of the literature revealed no studies published assessing the test-item-in-rotation dependence of any oblique machine methods. Only on varimax has such a determination been made. Cliff and Pennell (1967) seem to say that such determination should be made for other methods.

The six rotation methods chosen to be compared were selected for various reasons. Varimax was used because it is the only one known to the large majority of users and readers of factor analysis, and because it is the method of choice in a great many studies. Maxplane was selected because of its unique, almost non-analytic method of arriving at factor locations, and because of the high promise held out for it by Cattell (1966, p. 188). Oblimax was selected from Harman's (1967) potpourri of oblique multiple-factor methods, because its function involving the ratio of fourth powers of loadings to sums of squared loadings on a single factor was developed early and is still somewhat unique. The oblimin (biquartimin) procedure is based on the covariance of factor loadings and was the earliest to become generally available. It has proved satisfactory for a wide range of applications.

According to its developers, binormamin improves on oblimin by removing the arbitrary parameter. Jennrich and Sampson's (1966) simple loadings method seemed worthy of selection because application of the criterion to the P instead of the V matrix seems to be a theoretical breakthrough.

Significance of the Results

The study has shown that in a series of identical conditions of test battery composition for each method there is a significant difference among the six methods in the stability of rotated factor loadings and in the stability of primary factor intercorrelations. They do not perform equally well. There is also a substantial difference among the methods in their ability to achieve compelling and satisfying simple structure over all the test battery conditions.

It may be properly inferred that the orthogonal varimax and oblique simple loadings procedures are relatively insensitive to changes in test battery composition, even in situations where common-factor representation is reduced to the absolute minimum of only two weak tests on the weakest factor. This quality of robustness for these methods is particularly evident in the case of stability of factor loadings. The situation with respect to factor intercorrelations is less clear in that binormamin and perhaps oblimax also appear equally stable as simple loadings. Examination of the ability of all these methods to reach compelling and satisfying simple structure shows binormamin to be clearly superior.

The conditions imposed on the batteries carried into rotation by these methods in this study were designed to provide a full range of test communality and factor strength variation that would occur in

nature. Thus, the ability of a method to cope with this aggregate of situations may be generalizable to other problems, especially those in which on oblique factor solution is demanded.

However, the specific conclusions of this study are necessarily limited to these data. Just as developers of rotational procedures who used the Holzinger and Harman data applied their methods to other data to show general applicability, so must these results be verified. These six methods may act quite differently with factor matrices of fewer or more factors, more differing angular relations between factors, higher or lower overall communalities, and fewer or more variables or tests.

The results of this study have two broad implications. First, an investigator using the simple loadings method, for example, may now have a more secure and empirically based feeling that his results are perhaps more generalizable to the domain in question than if he had used a method shown to be relatively more dependent in its solution on the test item content of the battery rotated. Second, and perhaps more important for the methodology of factor analysis, it can be seen that any consideration of the influences on rotated factor loadings with the intent of describing the sampling distribution of them must take into account the analytic machine procedure being used for rotation.

BIBLIOGRAPHY

Carroll, J. B. An analytic solution for approximating simple structure in factor analysis. Psychometrika, 1953, 18, 23-28.

Carroll, J. B. Biquartimin criterion for rotation to oblique simple structure in factor analysis. Science, 1957, 1926, 1114-1115.

Cattell, R. B. Factor analysis: an introduction to essentials. I and II. Biometrics, 1965, 21, 190-215 and 405-435.

Cattell, R. B. (Ed.). Handbook of multivariate experimental psychology. Chicago: Rand McNally, 1966.

Cattell, R. B. and Muerle, J. L. The "maxplane" program for factor rotation to oblique simple structure. Educational and Psychological Measurement, 1960, 20, 569-590.

Cliff, N. and Hamburger, C. The study of sampling errors in factor analysis by means of artificial experiments. Mimeographed report, University of Southern California, 1966.

Cliff, N. and Pennell, R. The influence of communality, factor strength, and loading size on the sampling characteristics of factor loadings. Psychometrika, 1967, 32, 309-326.

Eber, H. W. Toward oblique simple structure: maxplane. Multivariate Behavioral Research, 1966, 1, 112-125.

Ferguson, G. A. The concept of parsimony in factor analysis. Psychometrika, 1954, 19, 281-290.

Fruchter, B. Introduction to factor analysis. New York: Van Nostrand, 1954.

Glass, G. V. and Taylor, P. A. Factor analytic methodology. Review of Educational Research, 1966, 36, 566-587.

Guilford, J. P. and Zimmerman, W. S. Some variable-sampling problems in rotation of axes in factor analysis. Psychological Bulletin, 1963, 60, 289-301.

Harman, H. H. Modern factor analysis. University of Chicago Press, 1967.

Holzinger, K. J. and Harman, H. H. Factor analysis. University of Chicago Press, 1941.

Horst, P. Factor analysis of data matrices. New York: Holt, Rinehart and Winston, 1965.

Jennrich, R. I. and Sampson, P. F. Rotation for simple loadings. Psychometrika, 1966, 31, 312-323.

Kaiser, H. F. The varimax criterion for analytic rotation in factor analysis. Psychometrika, 1958, 23, 187-200.

Kaiser, H. F. and Dickman, K. W. Analytic determination of common factors. American Psychologist, 1959, 14, 425.

Meredith, W. Notes on factorial invariance. Psychometrika, 1964, 29, 177-185.

Merrifield, P. R. and Cliff, N. Factor analytic methodology. Review of Educational Research, 1963, 33, 510-522.

Pinzka, C. and Saunders, D. R. Analytic rotation to simple structure, II: Extension to an oblique solution. Research Bulletin RB-54-31 (1954). Princeton, N. J.: Educational Testing Service.

Saunders, D. R. The rationale for an oblimax method of transformation in factor analysis. Psychometrika, 1961, 26, 317-324.

Steele, R. G. D. and Torrie, J. H. Principles and procedures of statistics. New York: McGraw-Hill, 1960.

Thompson, J. W. Meaningful and unmeaningful rotation of factors. Psychological Bulletin, 1962, 59, 211-223.

Thurstone, L. L. Multiple-factor analysis. University of Chicago Press, 1947.

APPENDIX

Table A - 1

VARIMAX REFERENCE FACTORS - 20 TEST BATTERY

Tests	Factors I	II	III	IV	
1	192	174	672	126	
5	761	217	164	125	
6	761	101	176	206	
7	830	168	144	074	
8	568	254	343	100	
9	812	015	162	248	
10	137	706	-039	230	
11	172	607	104	345	
12	029	682	252	080	
13	207	583	416	027	
14	214	167	052	494	
15	124	073	157	490	
16	116	090	431	409	
17	136	191	093	632	
18	018	262	355	514	
20	384	098	460	227	
21	184	372	430	228	
22	390	040	402	336	
23	390	197	555	184	
24	353	442	222	320	
Sum a_{ij}^2	3.670	2.390	2.190	2.000	10.250

Table A - 2

VARIMAX REFERENCE FACTORS - CONDITION 1

	Factors				
Tests	I	II	III	IV	
1	138	188	680	134	
7	814	179	202	089	
9	795	015	225	254	
10	129	690	-047	276	
12	001	683	234	122	
13	167	595	413	064	
14	204	140	067	505	
15	107	048	168	494	
16	079	077	439	411	
17	121	154	103	643	
18	-015	238	352	528	
20	345	102	487	232	
22	356	036	432	338	
23	343	206	579	195	
Sum a_{ij}^2	1.910	1.530	1.180	1.760	6.990

Table A - 3

VARIMAX REFERENCE FACTORS - CONDITION 2

Tests	Factors I	II	III	IV	
1	208	162	667	142	
5	769	215	156	080	
6	774	101	161	163	
7	835	166	136	024	
8	577	249	338	075	
9	828	015	143	203	
10	150	710	-036	207	
11	194	610	099	328	
12	037	678	262	079	
13	214	575	425	024	
17	177	199	066	622	
18	055	263	336	524	
22	416	037	383	328	
23	408	188	546	181	
Sum a_{ij}^2	3.440	1.990	1.470	1.060	7.960

Table A - 4

VARIMAX REFERENCE FACTORS - CONDITION 3

	Factors				
Tests	I	II	III	IV	
5	762	237	090	155	
6	757	130	141	231	
7	832	184	080	106	
8	574	332	249	112	
9	804	041	156	273	
11	168	601	-060	367	
12	042	724	041	091	
13	224	676	220	033	
14	197	162	031	504	
15	108	106	160	487	
20	386	219	419	218	
21	189	476	311	223	
22	385	145	389	329	
24	350	475	097	337	
Sum a_{ij}^2	3.410	2.120	640	1.130	7.310

Table A - 5

VARIMAX REFERENCE FACTORS - CONDITION 4

Tests	Factors I	II	III	IV	
6	775	083	038	234	
8	627	269	199	102	
11	187	597	034	367	
12	089	705	155	081	
14	205	149	058	503	
15	137	074	189	475	
16	189	130	439	359	
17	132	177	129	630	
18	076	292	377	476	
20	464	135	387	193	
21	266	411	360	198	
22	453	067	354	306	
23	492	246	458	143	
24	387	442	132	331	
Sum a_{ij}^2	2.040	1.540	1.090	1.740	6.420

Table A - 6

VARIMAX REFERENCE FACTORS - CONDITION 5

Tests	Factors I	II	III	IV	
5	771	190	118	157	
6	758	080	168	230	
8	595	272	284	078	
11	189	609	060	342	
12	083	714	134	041	
14	189	169	122	491	
15	101	102	242	454	
16	119	165	483	315	
20	401	155	459	156	
21	218	433	388	152	
22	392	090	438	273	
24	369	455	191	304	
Sum a_{ij}^2	2.130	1.490	1.050	970	5.640

Table A - 7

VARIMAX REFERENCE FACTORS - CONDITION 6

Tests	Factors I	II	III	IV	
1	182	179	668	152	
5	764	208	177	102	
7	830	161	162	044	
9	819	001	176	214	
10	157	693	-057	252	
12	033	682	236	119	
13	203	587	409	060	
16	127	077	416	424	
17	168	162	068	635	
18	038	244	329	538	
22	397	029	399	334	
23	386	195	555	196	
Sum a_{ij}^2	2.400	1.520	1.520	1.190	6.630

Table A - 8

VARIMAX REFERENCE FACTORS - CONDITION 7

Tests	Factors I	II	III	IV	
6	758	150	093	239	
8	591	270	288	098	
11	143	648	126	266	
14	180	239	048	478	
15	106	136	157	481	
20	419	118	420	232	
21	210	382	431	181	
24	343	483	211	276	
Sum a_{ij}^2	1.330	990	540	760	3.620

Table A - 9

VARIMAX REFERENCE FACTORS - CONDITION 8

Tests	Factors I	II	III	IV	
1	162	063	170	689	
7	813	192	035	210	
9	818	030	203	189	
10	105	699	258	067	
13	151	514	077	514	
17	166	154	636	082	
18	025	175	550	353	
23	366	116	207	586	
Sum a_{ij}^2	1.560	850	900	130	4.61

Table A - 10

MAXPLANE PRIMARY FACTORS - 20 TEST BATTERY

	Factors				
Tests	I	II	III	IV	
1	-009	018	780	-091	
5	744	077	104	-002	
6	730	-075	086	128	
7	838	034	086	-058	
8	489	120	353	-080	
9	785	-186	045	204	
10	077	692	-094	131	
11	057	522	037	259	
12	-088	675	322	-136	
13	076	534	519	-244	
14	109	001	-105	571	
15	-012	-107	029	567	
16	-079	-109	394	376	
17	-016	-011	-090	741	
18	-195	072	286	504	
20	240	-085	464	105	
21	017	244	462	066	
22	243	-171	354	278	
23	224	019	601	-003	
24	234	306	164	217	
Sum a_{ij}^2	1.633	2.112	2.303	1.791	7.839

FACTOR INTERCORRELATIONS

259	432	409
	316	488
		586

Table A - 11

MAXPLANE PRIMARY FACTORS - CONDITION 1

Tests	Factor I	II	III	IV	
1	-011	-029	813	-100	
7	873	062	-038	-037	
9	811	-198	083	155	
10	039	918	-636	360	
12	-116	842	-155	076	
13	066	643	142	-080	
14	076	066	-144	579	
15	-046	-086	094	535	
16	-106	-130	495	335	
17	-066	061	-109	744	
18	-243	120	254	522	
20	242	-117	520	073	
22	239	-207	463	221	
23	227	-001	591	-005	
Sum a_{ij}^2	1.633	2.112	2.303	1.791	7.839

FACTOR INTERCORRELATIONS

415	517	479	
	723	446	
		598	

Table A - 12

MAXPLANE PRIMARY FACTORS - CONDITION 2

Tests	Factors I	II	III	IV	
1	082	091	678	-089	
5	908	-053	-116	-010	
6	898	-226	-041	067	
7	999	-110	-120	-089	
8	628	069	133	-063	
9	963	-365	-023	101	
10	121	679	-552	400	
11	123	500	-334	469	
12	-053	759	-123	123	
13	151	610	112	-053	
17	056	-112	-123	768	
18	-134	069	207	561	
22	365	-233	346	210	
23	348	024	465	-009	
Sum a_{ij}^2	4.288	1.944	1.341	1.381	8.954

FACTOR INTERCORRELATIONS

524	568	519	
	583	514	
		612	

Table A - 13

MAXPLANE PRIMARY FACTORS - CONDITION 3

Tests	Factors I	II	III	IV	
5	853	111	-105	-017	
6	793	-033	021	039	
7	967	044	-114	-084	
8	439	219	242	-132	
9	842	-151	061	069	
11	165	655	-431	452	
12	-078	820	-179	086	
13	-013	695	169	-134	
14	133	104	-136	544	
15	-109	024	170	458	
20	013	070	648	-080	
21	-160	429	391	037	
22	035	-019	589	070	
24	257	442	-106	279	
Sum a_{ij}^2	3.349	2.067	1.311	857	7.584

FACTOR INTERCORRELATIONS

	469	800	493
		556	334
			674

Table A - 14

MAXPLANE PRIMARY FACTORS - CONDITION 4

Tests	Factors I	II	III	IV	
6	751	031	-009	127	
8	433	209	328	-129	
11	091	543	-192	460	
12	-122	682	151	014	
14	119	037	-129	594	
15	-070	-072	151	469	
16	-257	-058	687	118	
17	-035	021	-054	727	
18	-335	104	480	353	
20	088	-004	672	-122	
21	-116	292	560	-038	
22	109	-090	568	052	
23	043	104	815	-241	
24	220	361	051	288	
Sum a_{ij}^2	1.056	1.060	2.667	1.644	6.427

FACTOR INTERCORRELATIONS

205	692	426	
	414	329	
		752	

Table A - 15

MAXPLANE PRIMARY FACTORS - CONDITION 5

Tests	Factors I	II	III	IV	
5	704	075	105	083	
6	642	-056	167	121	
8	422	187	405	-103	
11	125	452	-072	488	
12	010	659	159	085	
14	047	-024	-001	552	
15	-124	-074	216	418	
16	-249	025	659	082	
20	084	054	675	-131	
21	-049	335	548	-017	
22	069	-048	600	022	
24	222	301	163	304	
Sum a_{ij}^2	1.245	897	1.857	874	4.873

FACTOR INTERCORRELATIONS

	032	597	362
		119	196
			717

Table A - 16

MAXPLANE PRIMARY FACTORS - CONDITION 6

Tests	Factors I	II	III	IV	
1	030	064	651	032	
5	770	115	-016	012	
7	855	066	026	-071	
9	832	-224	-060	295	
10	108	848	-553	158	
12	-072	859	-028	-133	
13	090	702	264	-246	
16	-002	-137	109	617	
17	086	-058	-574	999	
18	-105	073	-155	774	
22	309	-205	157	467	
23	270	058	436	117	
Sum a_{ij}^2	2.229	2.094	1.385	2.400	8.114

FACTOR INTERCORRELATIONS

	373	411	411
		616	717
			783

Table A - 17

MAXPLANE PRIMARY FACTORS - CONDITION 7

Tests	Factors I	II	III	IV	
6	912	-140	-132	094	
8	655	148	152	-237	
11	-027	811	-148	041	
14	109	008	-114	566	
15	023	-169	107	570	
20	433	-175	439	-006	
21	097	343	398	-154	
24	264	460	-004	038	
Sum a_{ij}^2	1.541	1.088	438	737	3.804

FACTOR INTERCORRELATIONS

	637	585	586
		719	777
			686

Table A - 18

MAXPLANE PRIMARY FACTORS - CONDITION 8

Tests	Factors I	II	III	IV	
1	067	104	672	100	
7	880	097	036	-148	
9	859	-155	-025	100	
10	065	673	048	113	
13	092	614	531	-111	
17	065	-095	-046	706	
18	-112	039	292	590	
23	302	092	510	111	
Sum a_{ij}^2	1.637	893	1.085	926	4.541

FACTOR INTERCORRELATIONS

227	327	399	
	119	513	
		197	

Table A - 19

OBLIMIN PRIMARY FACTORS - 20 TEST BATTERY

Tests	Factors I	II	III	IV	
1	109	082	643	031	
5	783	114	004	-023	
6	780	-022	022	080	
7	874	062	-013	-082	
8	552	163	224	-040	
9	840	-126	006	132	
10	056	727	-208	139	
11	071	586	-062	262	
12	-067	702	147	-029	
13	128	564	318	-114	
14	142	089	-078	484	
15	041	-012	063	499	
16	015	-012	368	386	
17	029	102	-047	642	
18	-116	181	259	500	
20	333	-014	383	140	
21	091	310	340	137	
22	336	-085	312	272	
23	328	088	476	070	
24	278	376	065	220	
Sum a_{ij}^2	3.489	2.053	1.396	1.596	8.543

FACTOR INTERCORRELATIONS

304	324	364	
	316	334	
		310	

Table A - 20

OBLIMIN PRIMARY FACTORS - CONDITION 1

Tests	Factors I	II	III	IV	
1	054	113	660	033	
7	822	107	080	-043	
9	785	-087	097	158	
10	060	694	-209	210	
12	-077	689	137	025	
13	096	576	332	-068	
14	128	067	-066	509	
15	023	-031	066	509	
16	-020	-012	371	388	
17	016	067	-047	667	
18	-139	158	246	518	
20	282	013	422	149	
22	287	-065	351	279	
23	271	120	514	085	
Sum a_{ij}^2	1.586	1.374	1.394	1.566	5.920

FACTOR INTERCORRELATIONS

208	266	319	
	275	307	
		347	

Table A - 21

OBLIMIN PRIMARY FACTORS - CONDITION 2

Tests	Factors I	II	III	IV	
1	143	077	629	067	
5	788	118	000	-045	
6	796	-011	018	053	
7	871	064	-018	-105	
8	564	161	217	-040	
9	860	-111	001	099	
10	074	731	-189	116	
11	109	597	-046	241	
12	-054	692	156	-015	
13	138	550	317	-091	
17	104	138	-034	601	
18	-044	203	264	491	
22	382	-070	305	266	
23	361	088	464	087	
Sum a_{ij}^2	3.420	1.808	985	792	7.005

FACTOR INTERCORRELATIONS

	286	281	269
		287	247
			185

Table A - 22

OBLIMIN PRIMARY FACTORS - CONDITION 3

Tests	Factors I	II	III	IV	
5	673	-377	-758	474	
6	665	-415	-576	411	
7	656	-488	-828	482	
8	666	-277	127	-409	
9	669	-485	-586	475	
11	573	489	-907	977	
12	487	443	-148	-054	
13	612	160	413	-746	
14	445	245	-568	928	
15	410	209	091	309	
20	616	-214	988	-999	
21	605	140	724	-773	
22	612	-177	796	-704	
24	658	186	-435	434	
Sum a_{ij}^2	4.978	1.560	5.621	5.958	18.117

FACTOR INTERCORRELATIONS

-047	005	035
	450	-527
		969

Table A - 23

OBLIMIN PRIMARY FACTORS - CONDITION 4

Tests	Factors I	II	III	IV	
6	818	-064	-212	071	
8	681	166	-016	-075	
11	044	570	-163	291	
12	012	737	017	-036	
14	086	047	-105	514	
15	041	-035	057	510	
16	156	009	306	352	
17	-026	061	-042	677	
18	-032	192	229	491	
20	509	006	214	090	
21	244	343	197	102	
22	470	-082	173	236	
23	550	125	270	007	
24	314	364	-088	224	
Sum a_{ij}^2	2.111	1.210	429	1.569	4.890

FACTOR INTERCORRELATIONS

376	221	504	
	264	397	
		236	

Table A - 24

OBLIMIN PRIMARY FACTORS - CONDITION 5

Tests	Factors I	II	III	IV	
5	825	057	-074	-013	
6	806	-074	-020	085	
8	611	166	141	-085	
11	071	595	-122	263	
12	-018	752	029	-073	
14	095	088	-034	490	
15	-001	019	126	471	
16	013	076	403	289	
20	371	041	362	051	
21	130	383	286	051	
22	349	-039	323	197	
24	292	387	012	198	
Sum a_{ij}^2	1.600	1.270	538	718	4.126

FACTOR INTERCORRELATIONS

	376	355	444
		307	350
			345

Table A - 25

OBLIMIN PRIMARY FACTORS - CONDITION 6

	Factors				
Tests	I	II	III	IV	
1	109	107	578	196	
5	778	126	020	-015	
7	861	078	016	-082	
9	840	-110	-006	127	
10	094	694	-209	132	
12	-047	687	141	049	
13	133	566	313	-002	
16	047	-006	262	478	
17	091	080	-159	639	
18	-065	170	142	577	
22	352	-071	238	345	
23	331	109	423	194	
Sum a_{ij}^2	2.341	1.372	846	1.208	5.767

FACTOR INTERCORRELATIONS

239	248	321	
	183	308	
		228	

Table A - 26

OBLIMIN PRIMARY FACTORS - CONDITION 7

Tests	Factors I	II	III	IV	
6	816	-025	-102	087	
8	605	145	148	-075	
11	000	651	-006	145	
14	078	143	-082	469	
15	-004	024	062	502	
20	385	-038	322	138	
21	104	311	353	069	
24	252	411	063	144	
Sum a_{ij}^2	1.261	734	275	571	2.841

FACTOR INTERCORRELATIONS

440	383	465	
	332	450	
		323	

Table A - 27

OBLIMIN PRIMARY FACTORS - CONDITION 8

Tests	Factors I	II	III	IV	
1	078	-036	689	082	
7	815	114	096	-070	
9	818	-067	073	123	
10	039	695	-045	182	
13	065	463	464	-040	
17	103	086	-017	628	
18	-067	096	296	518	
23	297	011	543	110	
Sum a_{ij}^2	1.448	733	1.089	736	4.006

FACTOR INTERCORRELATIONS

195	271	216	
	263	231	
		261	

Table A - 28

BINORMAMIN PRIMARY FACTORS - 20 TESTS

Tests	Factors I	II	III	IV	
1	-044	-005	794	-077	
5	772	110	044	-055	
6	761	-042	067	063	
7	873	059	027	-113	
8	493	134	302	-105	
9	823	-152	051	127	
10	061	780	-250	122	
11	038	605	-069	240	
12	-125	717	178	-107	
13	042	552	396	-219	
14	112	073	-086	515	
15	-016	-052	082	518	
16	-101	-086	453	351	
17	-021	075	-052	679	
18	-225	125	314	478	
20	231	-080	487	080	
21	-011	268	422	065	
22	241	-153	401	236	
23	207	018	600	-015	
24	226	368	096	186	
Sum a_{ij}^2	3.162	2.126	2.253	1.742	9.283

FACTOR INTERCORRELATIONS

365	523	454	
	515	467	
		535	

Table A - 29

BINORMAMIN PRIMARY FACTORS - CONDITION 1

Tests	Factors I	II	III	IV	
1	-033	046	764	-082	
7	849	101	045	-105	
9	798	-110	079	116	
10	075	740	-260	187	
12	-104	704	145	-049	
13	057	568	363	-175	
14	113	058	-070	519	
15	-012	-058	092	511	
16	-092	-067	446	339	
17	-014	051	-037	682	
18	-208	120	305	483	
20	229	-037	481	071	
22	237	-117	404	221	
23	208	066	584	-017	
Sum a_{ij}^2	1.595	1.438	1.855	1.497	6.385

FACTOR INTERCORRELATIONS

280	474	422	
	454	417	
		515	

Table A - 30

BINORMAMIN PRIMARY FACTORS - CONDITION 2

Tests	Factors I	II	III	IV	
1	-055	-027	763	016	
5	773	123	054	-078	
6	768	-035	070	042	
7	869	081	041	-144	
8	488	138	299	-088	
9	833	-147	053	101	
10	091	765	-243	109	
11	072	578	-075	248	
12	-119	704	174	-069	
13	035	546	390	-167	
17	035	021	-079	690	
18	-186	071	278	536	
22	255	-178	377	276	
23	203	002	578	047	
Sum a_{ij}^2	3.048	1.815	1.491	995	7.349

FACTOR INTERCORRELATIONS

	334	509	436
		496	506
			485

Table A - 31

BINORMAMIN PRIMARY FACTORS - CONDITION 3

Tests	Factors I	II	III	IV	
5	786	071	023	-034	
6	737	-081	106	057	
7	895	010	013	-097	
8	436	178	300	-125	
9	785	-203	126	108	
11	089	591	-226	346	
12	-093	786	-015	-027	
13	009	661	270	-197	
14	078	032	-062	546	
15	-115	-047	164	503	
20	070	016	589	-001	
21	-119	373	410	042	
22	076	-083	529	158	
24	212	377	027	233	
Sum a_{ij}^2	2.874	1.780	1.068	833	6.555

FACTOR INTERCORRELATIONS

	431	716	520
		529	465
			627

Table A - 32

BINORMAMIN PRIMARY FACTORS - CONDITION 4

Tests	Factors I	II	III	IV	
6	800	-072	-009	101	
8	530	157	234	-144	
11	094	613	-184	288	
12	-068	784	052	-126	
14	118	032	-065	532	
15	-033	-078	171	465	
16	-132	-064	589	183	
17	-026	031	011	667	
18	-243	142	419	348	
20	225	-051	548	-057	
21	001	320	431	-052	
22	227	-146	484	114	
23	207	070	646	-180	
24	268	375	023	188	
Sum a_{ij}^2	1.243	1.324	1.786	1.315	5.668

FACTOR INTERCORRELATIONS

	405	636	444
		597	513
			664

Table A - 33

BINORMAMIN PRIMARY FACTORS - CONDITION 5

Tests	Factors I	II	III	IV	
5	827	022	-035	005	
6	791	-127	037	095	
8	534	137	260	-157	
11	063	587	-168	322	
12	-069	781	050	-105	
14	071	029	-047	545	
15	-079	-046	176	464	
16	-163	010	581	157	
20	216	-012	555	-088	
21	-009	358	429	-063	
22	205	-108	495	087	
24	244	358	044	201	
Sum a_{ij}^2	1.790	1.261	1.211	745	5.007

FACTOR INTERCORRELATIONS

	451	638	508
		522	486
			692

Table A - 34

BINORMAMIN PRIMARY FACTORS - CONDITION 6

Tests	Factors I	II	III	IV	
1	-021	039	737	-028	
5	764	110	067	-051	
7	856	065	059	-117	
9	822	-150	069	110	
10	094	723	-254	178	
12	-099	699	151	-031	
13	058	559	369	-148	
16	-054	-081	396	379	
17	043	026	-092	694	
18	-161	106	251	519	
22	269	-140	368	249	
23	226	048	562	021	
Sum a_{ij}^2	2.169	1.405	1.459	1.041	6.074

FACTOR INTERCORRELATIONS

308	458	402	
	430	441	
		508	

Table A - 35

BINORMAMIN PRIMARY FACTORS - CONDITION 7

Tests	Factors I	II	III	IV	
6	844	-000	-064	030	
8	483	102	326	-178	
11	-005	711	002	025	
14	105	148	-124	478	
15	-070	-045	100	537	
20	147	-193	583	100	
21	-139	190	604	-021	
24	189	414	145	045	
Sum a_{ij}^2	1.038	785	861	563	3.247

FACTOR INTERCORRELATIONS

484	741	559	
	683	646	
		635	

Table A - 36

BINORMAMIN PRIMARY FACTORS - CONDITION 8

	Factors				
Tests	I	II	III	IV	
1	-013	-097	760	045	
7	824	118	062	-117	
9	828	-103	047	123	
10	009	765	-134	087	
13	-010	492	463	-153	
17	072	033	-062	663	
18	-140	037	296	523	
23	227	-038	582	073	
Sum a_{ij}^2	1.441	865	1.246	780	4.332

FACTOR INTERCORRELATIONS

323	445	328	
	481	493	
		437	

Table A - 37

OBLIMAX PRIMARY FACTORS - 20 TESTS

Tests	Factors I	II	III	IV	
1	-141	-132	972	-118	
5	825	116	-042	-069	
6	816	-041	-020	054	
7	937	064	-060	-133	
8	492	096	307	-130	
9	888	-151	-048	121	
10	073	873	-396	156	
11	033	665	-181	272	
12	-178	725	185	-108	
13	-018	512	460	-244	
14	133	120	-210	558	
15	-022	-040	018	554	
16	-157	-143	504	358	
17	-013	123	-181	735	
18	-280	107	313	506	
20	196	-152	549	060	
21	-069	222	474	056	
22	219	-208	425	231	
23	152	-071	695	-045	
24	223	388	024	200	
Sum a_{ij}^2	3.599	2.383	3.102	1.885	10.969

FACTOR INTERCORRELATIONS

453	674	536	
	668	504	
		670	

Table A - 38

OBLIMAX PRIMARY FACTORS - CONDITION 1

Tests	Factors I	II	III	IV	
1	-105	-056	907	-149	
7	892	127	-009	-116	
9	830	-061	-021	107	
10	129	750	-336	307	
12	-100	639	205	006	
13	044	482	466	-166	
14	132	112	-211	592	
15	-020	-024	-010	558	
16	-138	-093	446	337	
17	-004	109	-198	772	
18	-240	106	266	530	
20	196	-075	515	034	
22	210	-128	389	201	
23	167	002	659	-061	
Sum a_{ij}^2	1.729	1.293	2.399	1.867	7.288

FACTOR INTERCORRELATIONS

	185	596	504
		459	291
			688

Table A - 39

OBLIMAX PRIMARY FACTORS - CONDITION 2

Tests	Factors I	II	III	IV	
1	-148	-194	939	-006	
5	814	132	-045	-058	
6	807	-012	-016	050	
7	919	081	-070	-119	
8	482	080	291	-079	
9	879	-110	-041	104	
10	107	878	-360	121	
11	068	674	-144	243	
12	-160	677	185	-059	
13	-015	450	439	-156	
17	031	185	-116	644	
18	-244	117	349	489	
22	224	-200	430	248	
23	147	-106	675	032	
Sum a_{ij}^2	3.359	2.065	2.121	855	8.400

FACTOR INTERCORRELATIONS

	389	650	430
		668	350
			483

Table A - 40

OBLIMAX PRIMARY FACTORS - CONDITION 3

Tests	Factors I	II	III	IV	
5	845	086	-069	-035	
6	777	-090	066	050	
7	971	025	-098	-095	
8	448	158	260	-115	
9	823	-224	109	097	
11	076	662	-266	328	
12	-107	844	-099	-018	
13	-007	674	203	-176	
14	035	037	026	511	
15	-191	-081	311	473	
20	013	-062	677	001	
21	-180	341	460	046	
22	012	-163	650	150	
24	196	401	016	221	
Sum a_{ij}^2	3.265	2.012	1.410	733	7.420

FACTOR INTERCORRELATIONS

506	795	520	
	675	451	
		560	

Table A - 41

OBLIMAX PRIMARY FACTORS - CONDITION 4

Tests	Factors I	II	III	IV	
6	785	-155	067	124	
8	445	133	342	-163	
11	079	619	-238	314	
12	-167	852	066	-161	
14	170	-028	-138	599	
15	-003	-130	115	523	
16	-184	-084	610	198	
17	033	-030	-091	750	
18	-272	130	383	381	
20	139	-076	642	-068	
21	-096	339	485	-071	
22	177	-194	546	128	
23	084	065	767	-210	
24	232	357	027	206	
Sum a_{ij}^2	1.107	1.466	2.276	1.658	6.507

FACTOR INTERCORRELATIONS

513	673	425	
	660	619	
		733	

Table A - 42

OBLIMAX PRIMARY FACTORS - CONDITION 5

Tests	Factors I	II	III	IV	
5	945	-016	-152	-006	
6	906	-194	-034	079	
8	538	147	182	-151	
11	066	584	-158	287	
12	-161	871	-000	-106	
14	110	-061	060	501	
15	-100	-125	322	432	
16	-290	-012	722	153	
20	138	-015	586	-078	
21	-125	396	457	-059	
22	153	-148	564	084	
24	246	344	044	178	
Sum a_{ij}^2	2.258	1.476	1.583	632	5.949

FACTOR INTERCORRELATIONS

595	789	475	
	622	582	
		579	

Table A - 43

OBLIMAX PRIMARY FACTORS - CONDITION 6

Tests	Factors I	II	III	IV	
1	-076	-137	938	-144	
5	781	129	003	-022	
7	874	078	004	-094	
9	842	-092	-037	136	
10	130	821	-418	308	
12	-105	647	180	-000	
13	035	443	477	-170	
16	-074	-093	394	351	
17	073	199	-333	794	
18	-167	149	173	543	
22	255	-154	361	221	
23	192	-062	671	-049	
Sum a_{ij}^2	2.258	1.437	2.193	1.352	7.240

FACTOR INTERCORRELATIONS

	269	564	429
		564	293
			701

Table A - 44

OBLIMAX PRIMARY FACTORS - CONDITION 7

Tests	Factors I	II	III	IV	
6	942	-053	-134	035	
8	439	064	323	-147	
11	-074	827	-083	012	
14	165	163	-105	420	
15	-051	-070	216	478	
20	069	-285	741	106	
21	-297	183	719	-009	
24	141	456	117	039	
Sum a_{ij}^2	1.228	1.045	1.267	441	3.981

FACTOR INTERCORRELATIONS

649	845	443	
	794	609	
		504	

Table A - 45

OBLIMAX PRIMARY FACTORS - CONDITION 8

Tests	Factors I	II	III	IV	
1	-052	-174	853	005	
7	839	119	005	-098	
9	844	-088	011	121	
10	026	831	-258	132	
13	-030	457	449	-140	
17	085	111	-097	645	
18	-151	061	315	492	
23	202	-088	633	047	
Sum a_{ij}^2	1.491	975	1.505	722	4.693

FACTOR INTERCORRELATIONS

355	526	309	
	629	401	
		467	

Table A - 46

SIMPLE LOADINGS PRIMARY FACTORS - 20 TESTS

Tests	Factors I	II	III	IV	
1	079	097	644	034	
5	797	109	-014	-036	
6	791	-032	002	073	
7	891	056	-031	-098	
8	552	166	212	-050	
9	851	-140	-015	128	
10	059	725	-219	126	
11	065	582	-074	257	
12	-078	716	146	-043	
13	115	582	316	-129	
14	135	070	-095	495	
15	025	-029	050	515	
16	-012	-016	360	401	
17	014	079	-065	660	
18	-145	173	249	516	
20	317	-013	374	143	
21	071	316	334	136	
22	319	-091	300	280	
23	308	095	469	070	
24	271	372	051	216	
Sum a_{ij}^2	3.532	1.235	1.407	1.678	7.852

FACTOR INTERCORRELATIONS

333	382	417
	314	394
		345

Table A - 47

SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 1

Tests	Factors I	II	III	IV	
1	060	137	640	033	
7	860	115	006	-090	
9	818	-086	026	124	
10	069	697	-246	184	
12	-072	706	114	005	
13	109	600	298	-093	
14	132	056	-096	510	
15	022	-039	047	520	
16	-021	-008	354	401	
17	014	054	-074	677	
18	-145	159	230	532	
20	296	025	386	141	
22	299	-059	314	275	
23	286	139	475	073	
Sum a_{ij}^2	1.733	1.440	1.241	1.691	6.105

FACTOR INTERCORRELATIONS

243	345	395	
	284	367	
		364	

Table A - 48

SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 2

Tests	Factors I	II	III	IV	
1	061	055	653	081	
5	795	114	-007	-042	
6	795	-028	004	064	
7	885	066	-026	-105	
8	542	154	221	-036	
9	859	-133	-016	113	
10	087	720	-199	120	
11	096	568	-057	256	
12	-076	692	169	-017	
13	103	553	338	-094	
17	069	068	-065	642	
18	-110	142	254	528	
22	330	-111	301	291	
23	299	065	477	101	
Sum a_{ij}^2	3.331	1.729	1.049	912	7.021

FACTOR INTERCORRELATIONS

	302	402	360
		319	375
			292

Table A - 49

SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 3

Tests	Factors I	II	III	IV	
5	807	083	-030	-028	
6	780	-049	027	078	
7	910	021	-037	-092	
8	529	229	162	-045	
9	834	-164	042	130	
11	063	559	-188	291	
12	-074	771	-040	-014	
13	099	692	146	-111	
14	091	042	-066	513	
15	-046	-000	093	526	
20	240	118	367	141	
21	010	440	246	149	
22	235	015	327	280	
24	248	389	-020	239	
Sum a_{ij}^2	3.267	1.836	405	949	6.457

FACTOR INTERCORRELATIONS

404	429	486
	238	408
		266

Table A - 50

SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 4

Tests	Factors I	II	III	IV	
6	839	-081	-203	065	
8	680	170	-002	-091	
11	035	567	-148	273	
12	-012	756	039	-066	
14	086	031	-106	521	
15	030	-042	053	520	
16	125	023	307	357	
17	-035	046	-046	687	
18	-065	203	232	493	
20	491	019	221	086	
21	215	362	210	088	
22	456	-076	176	238	
23	525	146	282	-003	
24	307	362	-075	210	
Sum a_{ij}^2	2.062	1.265	435	1.592	5.357

FACTOR INTERCORRELATIONS

412	263	526	
	211	454	
		257	

Table A - 51

SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 5

Tests	Factors I	II	III	IV	
5	840	031	-082	-017	
6	819	-104	-034	089	
8	613	167	137	-089	
11	051	576	-140	272	
12	-037	771	030	-082	
14	078	052	-065	520	
15	-024	-002	096	504	
16	-016	085	379	317	
20	359	053	349	063	
21	108	401	276	058	
22	335	-040	303	218	
24	279	373	-005	208	
Sum a_{ij}^2	2.094	1.280	494	820	4.688

FACTOR INTERCORRELATIONS

	425	375	506
		261	433
			384

Table A - 52

SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 6

Tests	Factors I	II	III	IV	
1	081	126	607	097	
5	789	116	002	-034	
7	877	068	-002	-102	
9	851	-132	-006	114	
10	090	688	-250	164	
12	-062	703	115	023	
13	118	588	297	-061	
16	020	-016	296	438	
17	072	044	-142	672	
18	-096	155	167	561	
22	337	-082	263	303	
23	312	116	441	117	
Sum a_{ij}^2	2.374	1.411	932	1.129	5.846

FACTOR INTERCORRELATIONS

	286	336	376
		249	371
			351

Table A - 53

SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 7

Tests	Factors I	II	III	IV	
6	824	-035	-078	073	
8	570	174	165	-089	
11	-019	708	-033	062	
14	074	118	-093	468	
15	-031	-010	055	539	
20	320	-040	340	189	
21	029	351	349	073	
24	223	447	054	100	
Sum a_{ij}^2	1.164	871	286	578	2.899

FACTOR INTERCORRELATIONS

	501	448	514
		359	615
			293

Table A - 54

SIMPLE LOADINGS PRIMARY FACTORS - CONDITION 8

Tests	Factors I	II	III	IV	
1	034	-039	702	071	
7	833	115	043	-094	
9	836	-084	020	120	
10	035	714	-097	142	
13	035	487	448	-085	
17	095	055	-059	646	
18	-099	075	279	525	
23	268	006	534	097	
Sum a_{ij}^2	1.487	777	1.072	758	4.094

FACTOR INTERCORRELATIONS

257	391	283	
	340	366	
		350	

Table A - 55

VARIMAX ROTATION - FACTOR I

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	192	138	208				182		162
5	761		769	762		771	764		
6	761		774	757	775	758		758	
7	830	814	835	832			830		813
8	568		577	574	627	595		591	
9	812	795	828	804			819		818
10	137	129	150				157		105
11	172		194	168	187	189		143	
12	029	001	037	042	089	083	033		
13	207	167	214	224			203		151
14	214	204		197	205	189		180	
15	124	107		108	137	101		106	
16	116	079			189	119	127		
17	136	121	177		132		168		166
18	018	-015	055		076		038		025
20	384	345		386	464	401		419	
21	184			189	266	218		210	
22	390	356	416	385	453	392	397		
23	390	343	408		492		386		366
24	353			350	387	369		343	

Table A - 56

VARIMAX ROTATION - FACTOR II

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	174	188	162				179		063
5	217		215	237		190	208		
6	101		101	130	083	080		150	
7	168	179	166	184			161		192
8	254		249	332	269	272		270	
9	015	015	015	041			001		030
10	706	690	710				693		699
11	607		610	601	597	609		648	
12	682	683	678	724	705	714	682		
13	583	595	575	676			587		514
14	167	140		162	149	169		239	
15	073	048		106	074	102		136	
16	090	077			130	165	077		
17	191	154	199		177		162		154
18	262	238	263		292		244		175
20	098	102		219	135	155		118	
21	372			476	411	433		382	
22	040	036	037	145	067	090	029		
23	197	206	188		246		195		116
24	442			475	442	455		483	

Table A - 57

VARIMAX ROTATION - FACTOR III

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	672	680	667				668		170
5	164		156	090		118	177		
6	176		161	141	038	168		093	
7	144	202	136	080			162		035
8	343		338	249	199	284		288	
9	162	225	143	156			176		203
10	-039	-047	-036				-057		258
11	104		099	-060	034	060		126	
12	252	234	262	041	155	134	236		
13	416	413	425	220			409		077
14	052	067		031	058	122		048	
15	157	168		160	189	242		157	
16	431	439			439	483	416		
17	093	103	066		129		068		636
18	355	352	336		377		329		550
20	460	487		419	387	459		420	
21	430			311	360	388		431	
22	402	432	383	389	354	438	399		
23	555	579	546		458		555		207
24	222			097	132	191		211	

Table A - 58

VARIMAX ROTATION - FACTOR IV

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	126	134	142				152		689
5	125		080	155		157	102		
6	206		163	231	234	230		239	
7	074	089	024	106			044		210
8	100		075	112	102	078		098	
9	248	254	203	273			214		189
10	230	276	207				252		067
11	345		328	367	367	342		266	
12	080	122	079	091	081	041	119		
13	027	064	024	033			060		514
14	494	505		504	503	491		478	
15	490	494		487	475	454		481	
16	409	411			359	315	424		
17	632	643	622		630		635		082
18	514	528	524		476		538		353
20	227	232		218	193	156		232	
21	228			223	198	152		181	
22	336	338	328	329	306	273	334		
23	184	195	181		143		196		586
24	320			337	331	304		276	

Table A - 59

MAXPLANE ROTATION - FACTOR I

Conditions

Tests	All	1	2	3	4	5	6	7	8
1	-009	-011	082				030		067
5	744		908	853		704	770		
6	730		898	793	751	642		912	
7	838	873	999	967			855		880
8	489		628	439	433	422		655	
9	785	811	963	842			832		859
10	077	039	121				108		065
11	057		123	165	091	125		-027	
12	-088	-116	-053	-078	-122	010	-072		
13	076	066	151	-013			090		092
14	109	076		133	119	047		109	
15	-012	-046		-109	-070	-124		023	
16	-079	-106			-257	-249	-002		
17	-016	-066	056		-035		086		065
18	-195	-243	-134		-335		-105		-112
20	240	242		013	088	084		433	
21	017			-160	-116	-049		097	
22	243	239	365	035	109	069	309		
23	224	227	348		043		270		302
24	234			257	220	222		264	

Table A - 60

MAXPLANE ROTATION - FACTOR II

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	018	-029	091				064		104
5	077		-053	111		075	115		
6	-075		-226	-033	031	-056		-140	
7	034	062	-110	044			066		097
8	120		069	219	209	187		148	
9	-186	-198	-365	-151			-224		-155
10	692	918	679				848		673
11	522		500	655	543	452		811	
12	675	842	759	820	682	659	859		
13	534	643	610	695			702		614
14	001	066		104	037	-024		008	
15	-107	-086		024	-072	-074		-169	
16	-109	-130			-058	025	-137		
17	-011	061	-112		021		-058		-095
18	072	120	069		104		073		039
20	-085	-117		070	-004	054		-175	
21	244			429	292	335		343	
22	-171	-207	-233	-019	-090	-048	-205		
23	019	-001	024		104		058		092
24	306			442	361	301		460	

Table A - 61

MAXPLANE ROTATION - FACTOR III

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	780	813	678				651		672
5	104		-116	-105		105	-016		
6	086		-041	021	-009	167		-132	
7	086	-038	-120	-114			026		036
8	353		133	242	328	405		152	
9	045	083	-023	061			-060		-025
10	-094	-636	-552				-553		048
11	037		-334	-431	-192	-072		-148	
12	322	-155	-123	-179	151	159	-028		
13	519	142	112	169			264		531
14	-105	-144		-136	-129	-001		-114	
15	029	094		170	151	216		107	
16	394	495			687	659	109		
17	-090	-109	-123		-054		-574		-046
18	286	254	207		480		-155		292
20	464	520		648	672	675		439	
21	462			391	560	548		398	
22	354	463	346	589	568	600	157		
23	601	591	465		815		436		510
24	164			-106	051	163		-004	

Table A - 62

MAXPLANE ROTATION - FACTOR IV

Conditions

Tests	All	1	2	3	4	5	6	7	8
1	-091	-100	-089				032		100
5	-002		-010	-017		083	012		
6	128		067	039	127	121		094	
7	-058	-037	-089	-084			-071		-148
8	-080		-063	-132	-129	-103		-237	
9	204	155	101	069			295		100
10	131	360	400				158		113
11	259		469	452	460	488		041	
12	-136	076	123	086	014	085	-133		
13	-244	-080	-053	-134			-246		-111
14	571	579		544	594	552		566	
15	567	535		458	469	418		570	
16	376	335			118	082	617		
17	741	744	768		727		999		706
18	504	522	561		353		774		590
20	105	073		-080	-122	-131		-006	
21	066			037	-038	-017		-154	
22	278	221	210	070	052	022	467		
23	-003	-005	-009		-241		117		111
24	217			279	288	304		038	

Table A - 63

OBLIMIN ROTATION - FACTOR I

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	109	054	143				109		078
5	783		788	673		825	778		
6	780		796	665	818	806		816	
7	874	822	871	656			861		815
8	552		564	666	681	611		605	
9	840	785	860	669			840		818
10	056	060	074				094		039
11	071		109	573	044	071		000	
12	-067	-077	-054	487	012	-018	-047		
13	128	096	138	612			133		065
14	142	128		445	086	095		078	
15	041	023		410	041	-001		-004	
16	015	-020			156	013	047		
17	029	016	104		-026		091		103
18	-116	-139	-044		-032		-065		-067
20	333	282		616	509	371		385	
21	091			605	244	130		104	
22	336	287	382	612	470	349	352		
23	328	271	361		550		331		297
24	278			658	314	292		252	

Table A - 64

OBLIMIN ROTATION - FACTOR II

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	082	113	077				107		-036
5	114		118	-377		057	126		
6	-022		-011	-415	-064	-074		-025	
7	062	107	064	-488			078		114
8	163		161	-277	166	166		145	
9	-126	-087	-111	-485			-110		-067
10	727	694	731				694		695
11	586		597	489	570	595		651	
12	702	689	692	443	737	752	687		
13	564	576	550	160			566		463
14	089	067		245	047	088		143	
15	-012	-031		209	-035	019		024	
16	-012	-012			009	076	-006		
17	102	067	138		061		080		086
18	181	158	203		192		170		096
20	-014	013		-214	006	041		-038	
21	310			140	343	383		311	
22	-085	-065	-070	-177	-082	-039	-071		
23	088	120	088		125		109		011
24	376			186	364	387		411	

Table A - 65

OBLIMIN ROTATION - FACTOR III

Conditions

Tests	All	1	2	3	4	5	6	7	8
1	643	660	629				578		689
5	004		000	-758		-074	020		
6	022		018	-576	-212	-020		-102	
7	-013	080	-018	-828			016		096
8	224		217	127	-016	141		148	
9	006	097	001	-586			-006		073
10	-208	-209	-189				-209		-045
11	-062		-046	-907	-163	-122		-006	
12	147	137	156	-148	017	029	141		
13	318	332	317	413			313		464
14	-078	-066		-568	-105	-034		-082	
15	063	066		091	057	126		062	
16	368	371			306	403	262		
17	-047	-047	-034		-042		-159		-017
18	259	246	264		229		142		296
20	383	422		988	214	362		322	
21	340			724	197	286		353	
22	312	351	305	796	173	323	238		
23	476	514	464		270		423		543
24	065			-435	-088	012		063	

Table A - 66

OBLIMIN ROTATION - FACTOR IV

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	031	033	067				196		082
5	-023		-045	474		-013	-015		
6	080		053	411	071	085		087	
7	-082	-043	-105	482			-082		-070
8	-040		-040	-409	-075	-085		-075	
9	132	158	099	475			127		123
10	139	210	116				132		182
11	262		241	977	291	263		145	
12	-029	025	-015	-054	-036	-073	049		
13	-114	-068	-091	-746			-002		-040
14	484	509		928	514	490		469	
15	499	509		309	510	471		502	
16	386	388			352	289	478		
17	642	667	601		677		639		628
18	500	518	491		491		577		518
20	140	149		-999	090	051		138	
21	137			-773	102	051		069	
22	272	279	266	-704	236	197	345		
23	070	085	087		007		194		110
24	220			434	224	198		144	

Table A - 67

BINORMAMIN ROTATION - FACTOR I

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	-044	-033	-055				-021		-013
5	772		773	786		827	764		
6	761		768	737	800	791		844	
7	873	849	869	895			856		824
8	493		488	436	530	534		483	
9	823	798	833	785			822		828
10	061	075	091				094		009
11	038		072	089	094	063		-005	
12	-125	-104	-119	-093	-068	-069	-099		
13	042	057	035	009			058		-010
14	112	113		078	118	071		105	
15	-016	-012		-115	-033	-079		-070	
16	-101	-092			-132	-163	-054		
17	-021	-014	035		-026		043		072
18	-225	-208	-186		-243		-161		-140
20	231	229		070	225	216		147	
21	-011			-119	001	-009		-139	
22	241	237	255	076	227	205	269		
23	207	208	203		207		226		227
24	226			212	268	244		189	

Table A - 68

BINORMAMIN ROTATION - FACTOR II

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	-005	046	-027				039		-097
5	110		123	071		022	110		
6	-042		-035	-081	-072	-127		-000	
7	059	101	081	010			065		118
8	134		138	178	157	137		102	
9	-152	-110	-147	-203			-150		-103
10	780	740	765				723		765
11	605		578	591	613	587		711	
12	717	704	704	786	784	781	699		
13	552	568	546	661			559		492
14	073	058		032	032	029		148	
15	-052	-058		-047	-078	-046		-045	
16	-086	-067			-064	010	-081		
17	075	051	021		031		026		033
18	125	120	071		142		106		037
20	-080	-037		016	-051	-012		-193	
21	268			373	320	358		190	
22	-153	-117	-178	-083	-146	-108	-140		
23	018	066	002		070		048		-038
24	368			377	375	358		414	

Table A - 69

BINORMAMIN ROTATION - FACTOR III

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	794	764	763				737		760
5	044		054	023		-035	067		
6	067		070	106	-009	037		-064	
7	027	045	041	013			059		062
8	302		299	300	234	260		326	
9	051	079	053	126			069		047
10	-250	-260	-243				-254		-134
11	-069		-075	-226	-184	-168		002	
12	178	145	174	-015	052	050	151		
13	396	363	390	270			369		463
14	-086	-070		-062	-065	-047		-124	
15	082	092		164	171	176		100	
16	453	446			589	581	396		
17	-052	-037	-079		011		-092		-062
18	314	305	278		419		251		296
20	487	481		589	548	555		583	
21	422			410	431	429		604	
22	401	404	377	529	484	495	368		
23	600	584	578		646		562		582
24	096			027	023	044		145	

Table A - 70

BINORMAMIN ROTATION - FACTOR IV

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	-077	-082	016				-028		045
5	-055		-078	-034		005	-051		
6	063		042	057	101	095		030	
7	-113	-105	-144	-097			-117		-117
8	-105		-088	-125	-144	-157		-178	
9	127	116	101	108			110		123
10	122	187	109				178		087
11	240		248	346	288	322		025	
12	-107	-049	-069	-027	-126	-105	-031		
13	-219	-175	-167	-197			-148		-153
14	515	519		546	532	545		478	
15	518	511		503	465	464		537	
16	351	339			183	157	379		
17	679	682	690		667		694		663
18	478	483	536		348		519		523
20	080	071		-001	-057	-088		100	
21	065			042	-052	-063		-021	
22	236	221	276	158	114	087	249		
23	-015	-017	047		-180		021		073
24	186			233	188	201		045	

Table A - 71

OBLIMAX ROTATION - FACTOR I

Tests	Conditions								
	All	1	2	3	4	5	6	7	8
1	-141	-105	-148				-076		-052
5	825		814	845		945	781		
6	816		807	777	785	906		942	
7	937	892	919	971			874		839
8	492		482	448	445	538		439	
9	888	830	879	823			842		844
10	073	129	107				130		026
11	033		068	076	079	066		-074	
12	-178	-100	-160	-107	-167	-161	-105		
13	-018	044	-015	-007			035		-030
14	133	132		035	170	110		165	
15	-022	-020		-191	-003	-100		-051	
16	-157	-138			-184	-290	-074		
17	-013	-004	031		033		073		085
18	-280	-240	-244		-272		-167		-151
20	196	196		013	139	138		069	
21	-069			-180	-096	-125		-297	
22	219	210	224	012	177	153	255		
23	152	167	147		084		192		202
24	223			196	232	246		141	

Table A - 72

OBLIMAX ROTATION - FACTOR II

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	-132	-056	-194				-137		-174
5	116		132	086		-016	129		
6	-041		-012	-090	-155	-194		-053	
7	064	127	081	025			078		119
8	096		080	158	133	147		064	
9	-151	-061	-110	-224			-092		-088
10	873	750	878				821		831
11	665		674	662	619	584		827	
12	725	639	677	844	852	871	647		
13	512	482	450	674			443		457
14	120	112		037	-028	-061		163	
15	-040	-024		-081	-130	-125		-070	
16	-143	-093			-084	-012	-093		
17	123	109	185		-030		199		111
18	107	106	117		130		149		061
20	-152	-075		-062	-076	-015		-285	
21	222			341	339	396		183	
22	-208	-128	-200	-163	-194	-148	-154		
23	-071	002	-106		065		-062		-088
24	388			401	357	344		456	

Table A - 73

OBLIMAX ROTATION - FACTOR III

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	972	907	939				938		853
5	-042		-045	-069		-152	003		
6	-020		-016	066	067	-034		-134	
7	-060	-009	-070	-098			004		005
8	307		291	260	342	182		323	
9	-048	-021	-041	109			-037		011
10	-396	-336	-360				-418		-258
11	-181		-144	-266	-238	-158		-083	
12	185	205	185	-099	066	-000	180		
13	460	466	439	203			477		449
14	-210	-211		026	-138	060		-105	
15	018	-010		311	115	322		216	
16	504	446			610	722	394		
17	-181	-198	-116		-091		-333		-097
18	313	266	349		383		173		315
20	549	515		677	642	586		741	
21	474			460	485	457		719	
22	425	389	430	650	546	564	361		
23	695	659	675		767		671		633
24	024			016	027	044		117	

Table A - 74

OBLIMAX ROTATION - FACTOR IV

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	-118	-149	-006				-144		005
5	-069		-058	-035		-006	-022		
6	054		050	050	124	079		035	
7	-133	-116	-119	-095			-094		-098
8	-130		-079	-115	-163	-151		-147	
9	121	107	104	097			136		121
10	156	307	121				308		132
11	272		243	328	314	287		012	
12	-108	006	-059	-018	-161	-106	-000		
13	-244	-166	-156	-176			-170		-140
14	558	592		511	599	501		420	
15	554	558		473	523	432		478	
16	358	337			198	153	351		
17	735	772	644		750		794		645
18	506	530	489		381		543		492
20	060	034		001	-068	-078		106	
21	056			046	-071	-059		-009	
22	231	201	248	150	128	084	221		
23	-045	-061	032		-210		-049		047
24	200			221	206	178		039	

Table A - 75

DIRECT OBLIMIN ROTATION - FACTOR I

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	079	060	061				081		034
5	797		795	807		840	789		
6	791		795	780	839	819		824	
7	891	860	885	910			877		833
8	552		542	529	680	613		570	
9	851	818	859	834			851		836
10	059	069	087				090		035
11	065		096	063	035	051		-019	
12	-078	-072	-076	-074	-012	-037	-062		
13	115	109	103	099			118		035
14	135	132		091	086	078		074	
15	025	022		-046	030	-024		-031	
16	-012	-021			125	-016	020		
17	014	014	069		-035		072		095
18	-145	-145	-110		-065		-096		-099
20	317	296		240	491	359		320	
21	071			010	215	108		029	
22	319	299	330	235	456	335	337		
23	308	286	299		525		312		268
24	271			248	307	279		223	

Table A - 76

SIMPLE LOADINGS ROTATION - FACTOR II

		Conditions							
Tests	All	1	2	3	4	5	6	7	8
1	097	137	055				126		-039
5	109		114	083		031	116		
6	-032		-028	-049	-081	-104		-035	
7	056	115	066	021			068		115
8	166		154	229	170	167		174	
9	-140	-086	-133	-164			-132		-084
10	725	697	720				688		714
11	582		568	559	567	576		708	
12	716	706	692	771	756	771	703		
13	582	600	553	692			588		487
14	070	056		042	031	052		118	
15	-029	-039		-000	-042	-002		-010	
16	-016	-008			023	085	-016		
17	079	054	068		046		044		055
18	173	159	142		203		155		075
20	-013	025		118	019	053		-040	
21	316			440	362	401		351	
22	-091	-059	-111	015	-076	-040	-082		
23	095	139	065		146		116		006
24	372			389	362	373		447	

Table A - 77

SIMPLE LOADINGS ROTATION - FACTOR III

		Conditions							
Tests	All	1	2	3	4	5	6	7	8
1	644	640	653				607		702
5	-014		-007	-030		-082	002		
6	002		004	027	-203	-034		-078	
7	-031	006	-026	-037			-002		043
8	212		221	162	-002	137		165	
9	-015	026	-016	042			-006		020
10	-219	-246	-199				-250		-097
11	-074		-057	-188	-148	-140		-033	
12	146	114	169	-040	039	030	115		
13	316	298	338	146			297		448
14	-095	-096		-066	-106	-065		-093	
15	050	047		093	053	096		055	
16	360	354			307	379	296		
17	-065	-074	-065		-046		-142		-059
18	249	230	254		232		167		279
20	374	386		367	221	349		340	
21	334			246	210	276		349	
22	300	314	301	327	176	303	263		
23	469	475	477		282		441		534
24	051			-020	-075	-005		054	

Table A - 78

SIMPLE LOADINGS ROTATION - FACTOR IV

	Conditions								
Tests	All	1	2	3	4	5	6	7	8
1	034	033	081				097		071
5	-036		-042	-028		-017	-034		
6	073		064	078	065	089		073	
7	-098	-090	-105	-092			-102		-094
8	-050		-036	-045	-091	-089		-089	
9	128	124	113	130			114		120
10	126	184	120				164		142
11	257		256	291	273	272		062	
12	-043	005	-017	-014	-066	-082	023		
13	-129	-093	-094	-111			-061		-085
14	495	510		513	521	520		468	
15	515	520		526	520	504		539	
16	401	401			357	317	438		
17	660	677	642		687		672		646
18	516	532	528		493		561		525
20	143	141		141	086	063		189	
21	136			149	088	058		073	
22	280	275	291	280	238	218	303		
23	070	073	101		-003		117		097
24	216			239	210	208		100	

Table A - 79

MAXPLANE ROTATION PRIMARY FACTOR CORRELATIONS

	Conditions								
Factors	All	1	2	3	4	5	6	7	8
I-II	259	415	524	469	205	032	373	637	227
I-III	432	517	568	800	692	597	411	585	327
I-IV	409	479	519	493	426	362	411	586	399
II-III	316	723	583	556	414	119	616	719	119
II-IV	488	446	514	334	329	196	717	777	513
III-IV	586	598	612	674	752	717	783	686	197

Table A - 80

OBLIMIN ROTATION PRIMARY FACTOR CORRELATIONS

	Conditions								
Factors	All	1	2	3	4	5	6	7	8
I-II	304	208	286	-047	376	376	239	440	195
I-III	324	266	281	005	221	355	248	383	271
I-IV	364	319	269	035	504	444	321	465	216
II-III	316	275	287	450	264	307	183	332	263
II-IV	334	307	247	-527	397	350	308	450	231
III-IV	310	347	185	969	236	345	228	323	261

Table A - 81

BINORMAMIN PRIMARY FACTOR CORRELATIONS

	Conditions								
Factors	All	1	2	3	4	5	6	7	8
I-II	365	280	334	431	405	451	308	484	323
I-III	523	474	509	716	636	638	458	741	445
I-IV	454	422	436	520	444	508	402	559	328
II-III	515	454	496	529	597	522	430	683	481
II-IV	467	417	506	465	513	486	441	646	493
III-IV	535	515	485	627	664	692	508	635	437

Table A - 82

OBLIMAX PRIMARY FACTOR CORRELATIONS

	Conditions								
Factors	All	1	2	3	4	5	6	7	8
I-II	453	185	389	506	513	595	269	649	355
I-III	674	596	650	795	673	789	564	845	526
I-IV	536	504	430	520	425	475	429	443	309
II-III	668	459	668	675	660	622	564	794	629
II-IV	504	291	350	451	619	582	293	609	401
III-IV	670	688	483	560	733	579	701	504	467

Table A - 83

SIMPLE LOADINGS PRIMARY FACTOR CORRELATIONS

	Conditions								
Factors	All	1	2	3	4	5	6	7	8
I-II	333	243	302	404	412	425	286	501	257
I-III	382	345	402	429	263	375	336	448	391
I-IV	417	395	360	486	526	506	376	514	283
II-III	314	284	319	238	211	261	249	359	340
II-IV	394	367	375	408	454	433	371	615	366
III-IV	345	364	292	266	257	384	351	293	350

Table A - 84

CODED SUMMARY OF ROTATED FACTOR LOADINGS - 20-TEST BATTERY

METHOD	FREQUENCY OF OCCURANCE				
	Blank	+	++	+++	++++
Varimax	47	11	13	7	2
Maxplane	55	7	9	8	1
Oblimin	51	14	7	6	2
Binormamin	55	6	9	8	2
Oblimax	53	7	10	4	6
Simple loadings	52	12	8	6	2

Absolute values of loadings are coded:

.000-.259 Blank

.260-.399 +

.400-.599 ++

.600-.799 +++

.800-.999 ++++

Table A - 85

CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 1

METHOD	FREQUENCY OF OCCURANCE				
	Blank	+	++	+++	++++
Varimax	34	3	13	5	1
Maxplane	39	2	7	3	5
Oblimin	36	8	6	5	1
Binormamin	38	4	8	5	1
Oblimax	37	5	7	4	3
Simple loadings	36	8	5	5	2

Absolute values of loadings are coded:

.000-.259 Blank

.260-.399 +

.400-.599 ++

.600-.799 +++

.800-.999 ++++

Table A - 86

CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 2

METHOD	FREQUENCY OF OCCURENCE				
	Blank	+	++	+++	++++
Varimax	33	7	7	7	2
Maxplane	35	5	6	6	4
Oblimin	37	6	5	6	2
Binormamin	38	5	5	6	2
Oblimax	38	3	5	4	6
Simple loadings	38	5	5	6	2

Absolute values of loadings are coded:

.000-.259 Blank

.260-.399 +

.400-.599 ++

.600-.799 +++

.800-.999 ++++

Table A - 87

CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 3

METHOD	FREQUENCY OF OCCURANCE				
	Blank	+	++	+++	++++
Varimax	33	10	6	5	2
Maxplane	38	2	8	4	4
Oblimin	11	3	20	16	6
Binormamin	38	5	7	5	1
Oblimax	37	5	5	5	4
Simple loadings	40	5	5	3	3

Absolute values of loadings are coded:

.000-.259 Blank

.260-.399 +

.400-.599 ++

.600-.799 +++

.800-.999 ++++

Table A - 88

CODED SUMMARY OF ROTATED FACTOR LANDINGS - CONDITION 4

METHOD	FREQUENCY OF OCCURANCE				
	Blank	+	++	+++	++++
Varimax	29	12	11	4	0
Maxplane	36	6	8	5	1
Oblimin	38	7	7	3	1
Binormamin	38	5	8	4	1
Oblimax	37	7	5	6	1
Simple loadings	38	7	7	3	1

Absolute values of loadings are coded:

.000-.259 Blank

.260-.399 +

.400-.599 ++

.600-.799 +++

.800-.999 ++++

Table A - 89

CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 5

METHOD	FREQUENCY OF OCCURANCE				
	Blank	+	++	+++	++++
Varimax	26	9	9	4	0
Maxplane	32	3	8	5	0
Oblimin	30	10	4	2	2
Binormamin	33	4	8	2	1
Oblimax	32	5	7	1	3
Simple loadings	30	10	4	2	2

Absolute values of loadings are coded:

.000-.259 Blank

.260-.399 +

.400-.599 ++

.600-.799 +++

.800-.999 ++++

Table A- 90

CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 6

METHOD	FREQUENCY OF OCCURANCE				
	Blank	+	++	+++	++++
Varimax	30	5	6	5	2
Maxplane	30	4	4	5	5
Oblimin	32	5	4	5	2
Binormamin	33	5	3	5	2
Oblimax	31	5	4	4	4
Simple loadings	31	6	4	5	2

Absolute values of loadings are coded:

.000-.259 Blank

.260-.399 +

.400-.599 ++

.600-.799 +++

.800-.999 ++++

Table A - 91

CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 7

METHOD	FREQUENCY OF OCCURANCE				
	Blank	+	++	+++	++++
Varimax	18	5	7	2	0
Maxplane	21	3	5	1	2
Oblimin	22	4	3	2	1
Binormamin	23	1	5	2	1
Oblimax	21	3	4	2	2
Simple loadings	22	4	4	1	1

Absolute values of loadings are coded:

.000-.259	Blank
.260-.399	+
.400-.599	++
.600-.799	+++
.800-.999	++++

Table A - 92

CODED SUMMARY OF ROTATED FACTOR LOADINGS - CONDITION 8

METHOD	FREQUENCY OF OCCURANCE				
	Blank	+	++	+++	++++
Varimax	21	2	4	3	2
Maxplane	21	2	3	4	2
Oblimin	21	2	4	3	2
Binormamin	22	1	4	3	2
Oblimax	22	1	3	2	4
Simple loadings	21	2	4	3	2

Absolute values of loadings are coded:

.000-.259 Blank

.260-.399 +

.400-.599 ++

.600-.799 +++

.800-.999 ++++

Table A - 93

MEANS, STANDARD DEVIATIONS, AND RELIABILITY COEFFICIENTS

TWENTY-FOUR PSYCHOLOGICAL TESTS*

Test	Mean	Standard Deviation	Reliability Coefficient
1	29.60	6.90	.756
2	24.84	4.50	.568
3	15.65	3.07	.544
4	36.31	8.38	.922
5	44.92	11.75	.808
6	9.95	3.36	.651
7	18.79	4.63	.754
8	28.18	5.34	.680
9	17.24	7.89	.870
10	90.16	23.60	.952
11	68.41	16.84	.712
12	109.83	21.04	.937
13	191.81	37.03	.889
14	176.14	10.72	.648
15	89.45	7.57	.507
16	103.43	6.74	.600
17	7.15	4.57	.725
18	9.44	4.49	.610
19	15.24	3.58	.569
20	30.38	19.76	.649
21	14.46	4.82	.784
22	27.43	9.77	.787
23	18.82	9.35	.931
24	25.83	4.70	.836

*Holzinger and Harman, 1941

BIOGRAPHICAL SKETCH

John Pelham Bailey, Jr., was born July 17, 1933, in Washington, D. C. In June, 1951, he was graduated from McDonogh School, Baltimore, Maryland, and won a full scholarship to attend Dartmouth College. In June, 1955, he received the degree of Bachelor of Arts from Dartmouth and was commissioned as Ensign in the regular Navy. Mr. Bailey remained in the Navy until 1964, serving in a wide variety of shipboard billets, staff billets, and as a mine warfare instructor in the Belgian Navy for two years. In 1964 he resigned his commission in the regular Navy and enrolled at the University of Florida as a graduate student in elementary education. Mr. Bailey received the degree of Master of Education in December, 1965. He worked as a graduate assistant for three years and as a research associate for one year in the College of Health Related Professions, and as a research associate in the College of Education for one year.

John Pelham Bailey, Jr., is married to the former Sara Anne Greeley and is the father of three children. He is a member of the Association for Computing Machinery, Phi Delta Kappa, American Educational Research Association, Phi Kappa Psi, and the Ready Reserve of the United States Navy.

This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Education and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

June, 1969

B Sharf by MC Beller

Dean, College of Education

Dean, Graduate School

Supervisory Committee:

Wilma H. Gustin

Chairman

R E Jester

B. N. Cage

Ben Barger